





# 计算多孔多相介质和颗粒材料力学

工业装备结构分析国家重点实验室  
大连理工大学运载工程与力学学部工程力学系

李锡夔

# 前言

- 目的：通过具体的研究工作进展介绍尽可能较广泛地了解“计算多孔多相介质和颗粒材料力学”领域的研究内容；通过所提供文献可更广泛地查阅到国内外在该领域的工作。
- 学科关联：  
计算多孔多相介质和颗粒材料力学  计算力学   
基于模拟的工程与科学（数学模型、数值方法、本构模拟）
- 内容说明：  
为介绍具体研究工作（思想、概念、方法）和说明问题，在报告中不得不列出一些数学公式。在ppt文件中引用的论文提供了细节。

# 内容提要-I

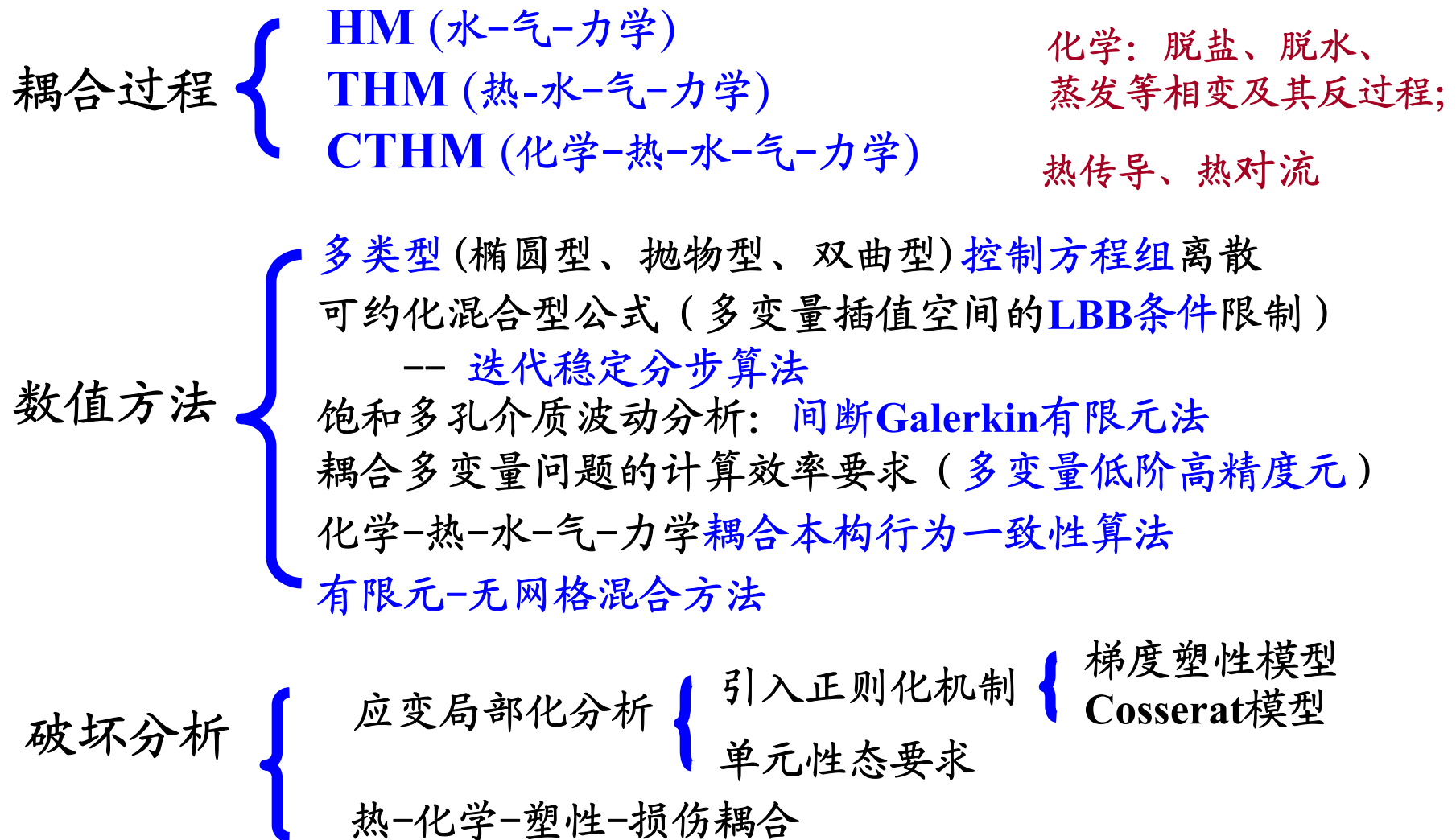
- 多孔介质与颗粒材料：组成与模型化
- 基于模拟的工程与科学(SBES)
  - 力学及耦合问题：多学科、多物理、多尺度
- 含孔隙流体多孔连续介质中渗流-力学耦合过程(HM)
  - 基本数学模型与数值方法
  - 压力稳定迭代分步算法
  - 梯度塑性和线性互补模型及有限元-无网格耦合方法
  - 间断 Galerkin 有限元法
- 多孔多相连续介质中热-渗流(湿、气)-力学耦合分析 (THM)的混合有限元方法

## 内容提要-II

- 高温下混凝土化学 - 热 - 湿 - 气 - 力学 (CTHM) 全耦合分级数学模型、本构模拟与数值方法
- 非饱和土中热 - 湿 - 气 - 力学 - 传质 (CTHM) 全耦合数学模型与数值方法
- 离散颗粒模型及离散元方法
  - 计及颗粒间滚动及滚动矩、颗粒分级破碎的离散颗粒模型
  - 含液颗粒材料液固耦合分析的离散-连续模型及特征线SPH法
- 颗粒材料多尺度方法
  - “离散颗粒集合体-梯度增强Cosserat多孔连续体”均匀化方法
  - “离散颗粒集合体-Cosserat多孔连续体”连接尺度方法

# 内容提要III: 按模型化-1

## 宏观模型: 多孔多相介质, 多孔连续体模型



# 内容提要III: 按模型化-2

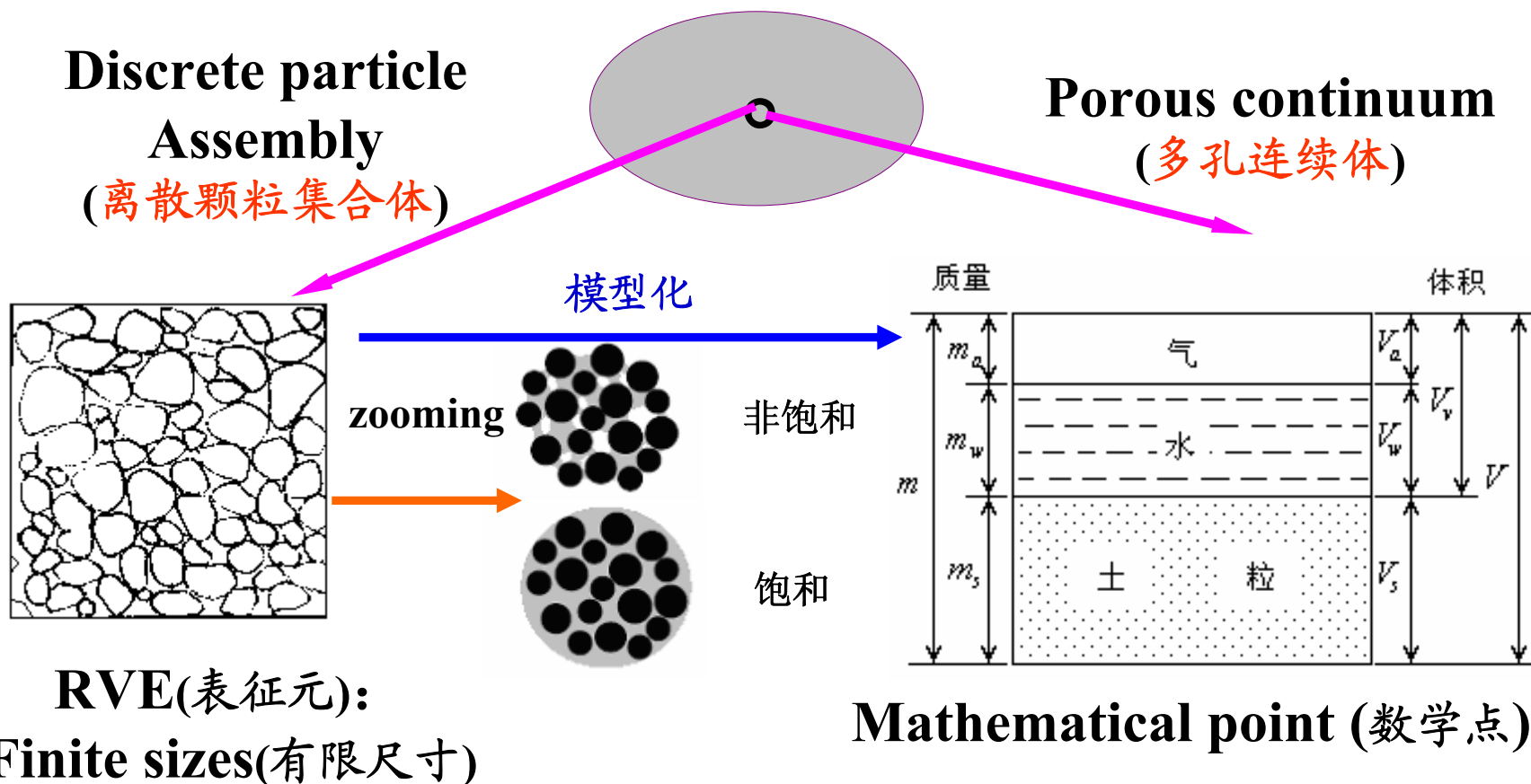
## 细观模型: 颗粒材料, 离散颗粒模型

- 离散接触颗粒运动学-动力学量本构模型
- 颗粒材料破坏模式
- 饱和离散颗粒集合体模型与特征基SPH方法

## 多尺度模型: 离散颗粒-多孔连续体模型

- “离散颗粒集合体-Cosserat多孔连续体”细宏观均匀化方法
- “离散颗粒集合体-梯度增强Cosserat多孔连续体”  
细宏观均匀化方法
- 颗粒材料的连接尺度方法

# 多孔介质与颗粒材料：组成与模型化



工程材料实例：岩土，混凝土，地质体，生物材料，粉末合金，泡沫材料等  
研究内容：力学及耦合(孔隙流动、传热与传质等问题) (多学科、多物理)

研究方法：数学模型、数值方法、本构模拟

→ “基于数值模拟的工程与科学” (SBES)

# 基于模拟的工程与科学(SBES)



## International Assessment of Research and Development in Simulation-based Engineering and Science

**National Science Foundation**

**Department of Defense**

**Department of Energy**

**National Aeronautics and Space Administration (NASA)**

**(美国)国家航空和宇宙航行局**

**National Institute of Health**

美国联合调查组访问中国(2007,12), 日本(2007,12), 欧洲 (2008,2)



# A recent report by the NSF Blue Ribbon Panel on SBES (Simulation Based Engineering & Science)

- Background

The use of **computer simulation** in engineering systems become an essential scientific methodology for research and **education** in **nearly all areas of engineering** and in **many branches of science**.

在工程系统中利用计算机模拟成为一个在几乎所有工程领域和许多科学分支中从事研究和教学的重要科学方法。

**Almost in all areas of engineering and science, computer simulation** has enabled the researchers to study and predict the physical events, as an extension of their **theoretical investigations**. In many cases, it also provides **a powerful alternative to the experimental science** when phenomena are not observable or measurements are **impractical or too expensive**.

几乎在所有的工程和科学领域，计算机模拟已使研究者，作为他们理论研究的延伸，能研究和预测物理事件。在许多情况下，当（物理）现象不可观察或对它们的测量是不切实际的或过于昂贵，它也提供了替代实验科学的强有力选择。

## The report concludes that

“**SBES is indispensable**

**to the nation’s continued leadership in science and engineering”**

(SBES 对于国家在科学与工程持续领先地位是不可或缺的)

and that “**computer simulation is central to advances in**

**biomedicine, manufacturing, homeland security,**

**microelectronics, energy and environmental sciences, advanced materials, and product development”.**

(计算机模拟对于生物医学、制造、国土安全、微电子、能源和环境科学、先进材料和产品研发的进展起着中心作用)

**In short, future advances in SBES research and education will**

**significantly impact virtually every aspect of human experience.**

简言之，今后在SBES研究和教学的进展事实上将从根本上冲击人类实践的每个方面。

**Chaired by J. Tinsley Oden, 2006 report on “Simulation-based Engineering and Science”:**

[www.nsf.gov/publications/pub\\_summ.jsp?ods\\_key=sbes0506](http://www.nsf.gov/publications/pub_summ.jsp?ods_key=sbes0506)

**“Advances**

- **in mathematical modeling,**
- **in computational algorithms,**
- **in the speed of computers,**
- **in the science and technology of data intensive computing**

**have brought the field of computation simulation to the threshold of a new era, (已把计算模拟带到了一个新世纪)**

**an era in which unprecedented improvements in the health, security, productivity, and competitiveness of our nation may be possible.**

**A host of critical technologies are on the horizon that cannot be understood, developed, or utilized without simulation methods”**

**(在这个新世纪中我们国家(美国)可能在健康、安全、生产率和竞争性能得到空前改善。但没有模拟方法,许多正在出现的关键技术将不能得到理解,发展或利用。)**

# “基于模拟的工程和科学” (SBES) (Simulation-based Engineering and Science)

(数学模型 (mathematical modeling) 的特点)

- 多学科交叉领域 (Multi-discipline)
- 多物理 (也包含化学、生物等) 过程 (Multi-physics)
- 多尺度的视角和相应的模型 (Multi-scale)

# 含液多孔连续介质中渗流-力学耦合过程 (HM)

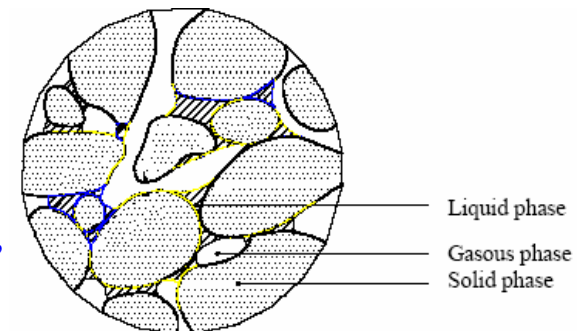
## 多孔多相连续介质：基本数学模型与数值方法

- 不混溶三相模型：固相、(混合)液相、(混合)气相
  - 两流相间存在毛细力（吸力）
  - 两流相与固相间存在渗流阻尼力
  - 采用Lagrangian坐标与Lagrangian-Eulerian坐标分别描述固相与流相的运动
  - 基于动量与质量守恒的控制方程
  - 基于虚功与余虚功原理的Galerkin弱形式( $u-p-S$ 公式)
  - 时域无条件稳定的积分求解过程:直接算法与交错算法
  - 算法的稳定性分析
- 物理模型
- 数学模型
- 数值方法与算法

Li Xikui, Zienkiewicz OC, Xie YM. A numerical model for immiscible two-phase fluid flow in a porous medium and its time domain solution, **Int. J. Numerical Methods in Eng.**, 1990, 30(6): 1195-1212.

Li Xikui. Finite element analysis for immiscible two-phase fluid flow in deforming porous media and an unconditionally stable staggered solution, **Communications in Applied Numerical Methods**, 1990, 6(2): 125-135.

Li Xikui, Zienkiewicz OC. Multiphase flow in deforming porous media and finite element solutions, **Computers & Structures**, 1992, 45(2): 211-227.



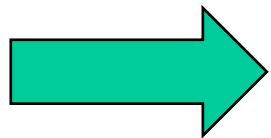
# 压力稳定迭代分步算法

## 问题的提出

- ▶ 饱和土u-p模型空间离散导致u-p混合公式;
- ▶ 在假定固体颗粒和孔隙水不可压缩, 以及渗透很低、因而也假定不排水条件下, u-p混合公式成为可约化;

$$\begin{bmatrix} \mathbf{K} & \mathbf{Q} \\ \mathbf{Q}^T & \mathbf{0} \end{bmatrix} \begin{Bmatrix} \mathbf{u} \\ \mathbf{p} \end{Bmatrix} = \begin{Bmatrix} \mathbf{R}_u \\ \mathbf{R}_p \end{Bmatrix}$$

- ▶ 满足LBB条件的u-p插值空间排除了采用等低阶u-p插值 (单元); (压力场虚假数值振荡)
- ▶ 现代数值分析要求采用低阶高精度插值近似;



Chorin (1968) 在流体力学领域提出了分步算法;

Pastor (1999) 将分步算法拓展应用于土动力学;

# 原有分步算法的压力稳定性问题 (流体力学, 饱和土力学)

$$\Psi_F = \frac{\Delta t}{\rho} (\mathbf{K}_p - \mathbf{L}\tilde{\mathbf{M}}^{-1}\mathbf{L}^T) (\bar{\mathbf{p}}^{n+1} - \gamma\bar{\mathbf{p}}^n)$$

- (1) 最大时间步长限制;    → 原有分步算法的显式时域积分
  - (2) 最小时间步长要求;
  - (3) 必须采用压力全量的分步算法;
- } 分步算法的稳定性条件

## 提出了压力稳定迭代分步算法

对于(1): 引入迭代过程 → 迭代稳定分步算法

- Xikui Li, Xianhong Han, M.Pastor. An **iterative stabilized fractional step algorithm for finite element analysis in saturated soil dynamics**,  
Computer Methods in Appl. Mech. Eng., 2003, 192:3845-3859.

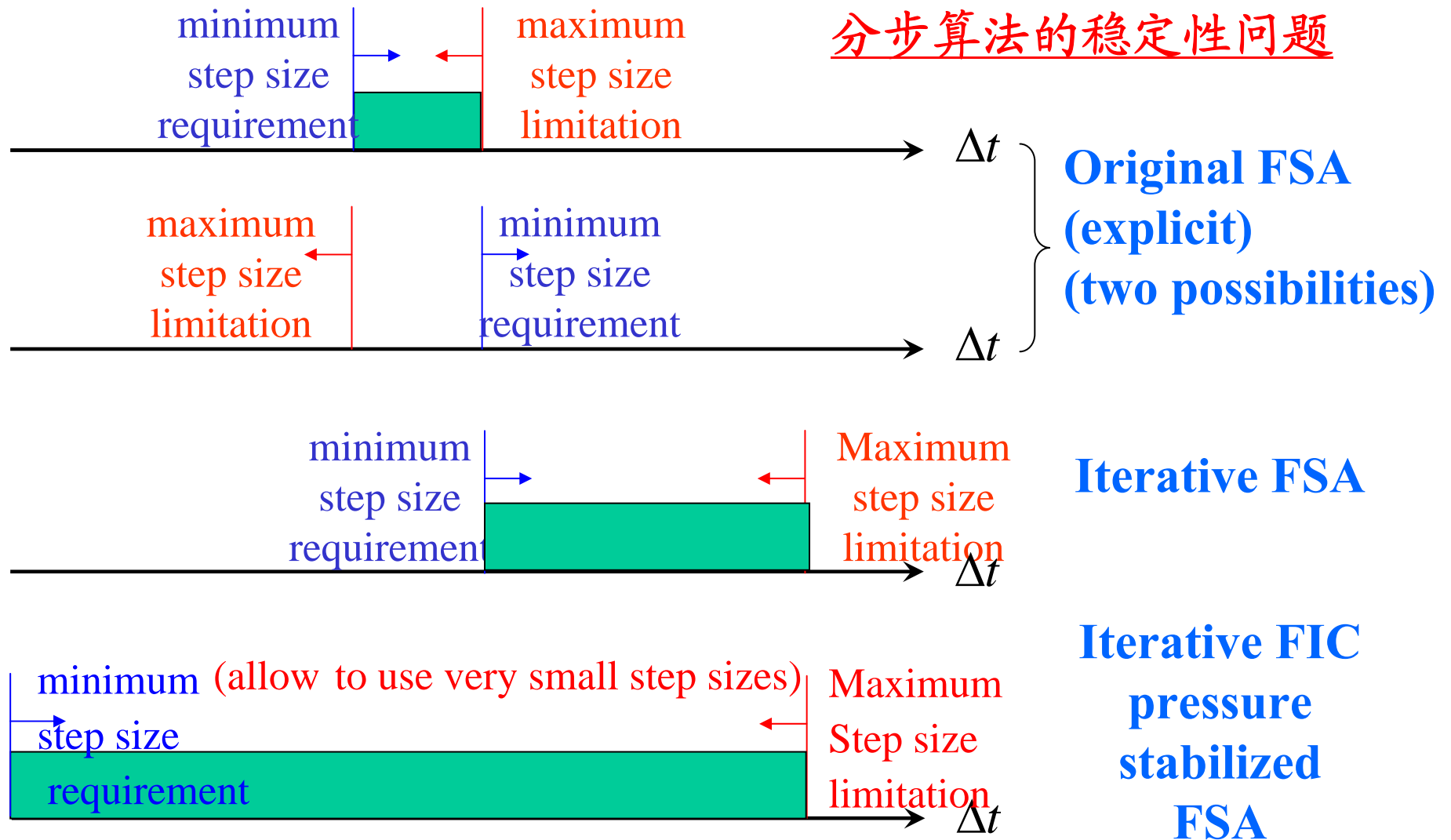
对于(2),(3): 引入FIC过程 → 压力稳定迭代分步算法

$$\Psi_S = \frac{\lambda^d}{\rho} (\mathbf{K}_p - \mathbf{D}\mathbf{M}^{-1}\mathbf{D}^T) \bar{\mathbf{p}}^{n+1}$$

FIC:  
Finite  
Increment  
Calculus

- Xikui Li, Xue Zhang, Xianhong Han, D.C.Sheng  
**An iterative pressure-stabilized fractional step algorithm in saturated soil dynamics**  
Int. J. Numerical and Analytical Methods in Geomechanics, 34:733-753, 2009.
- Xikui Li, Qinglin Duan  
**Meshfree iterative stabilized TGBS/CBS fractional step algorithms for incompressible N-S equations**,  
Computer Methods in Appl. Mech. Eng., 2006, 195: 6125-6145.

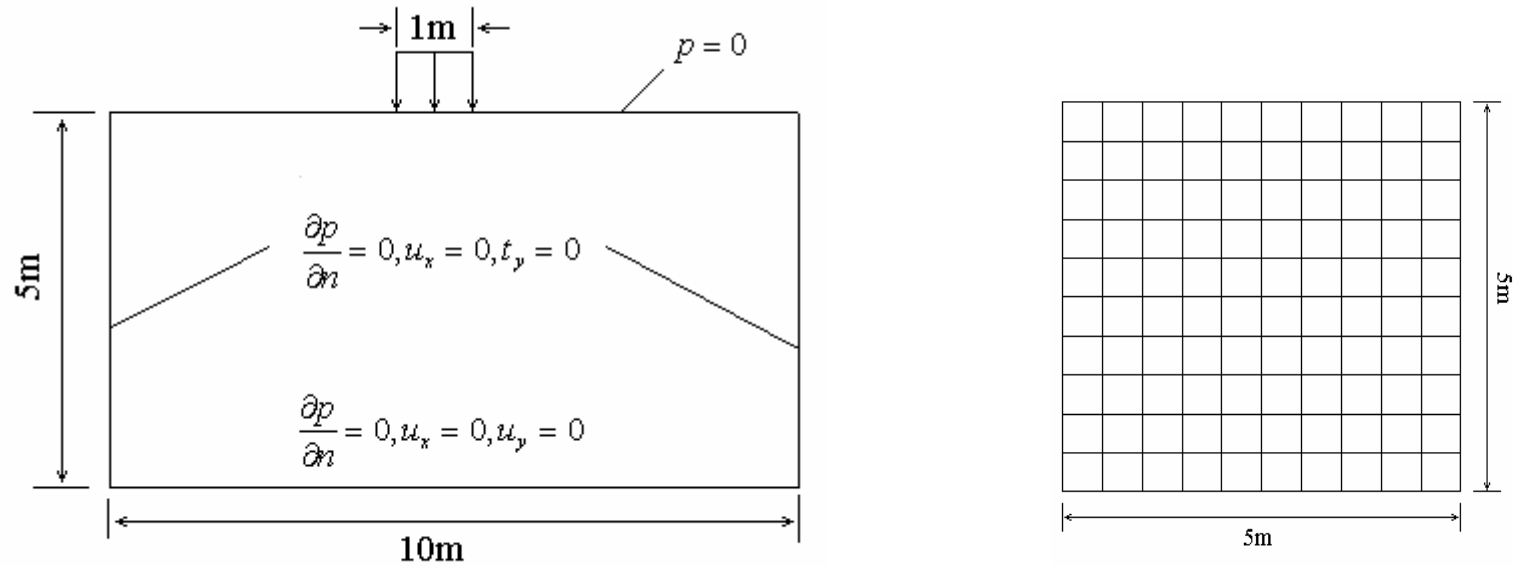
# 压力稳定迭代分步算法



- Xikui Li, Xue Zhang, Xianhong Han, D.C.Sheng. **An iterative pressure-stabilized fractional step algorithm in saturated soil dynamics**, *Int. J. Numerical and Analytical Methods in Geomechanics*, 2010, 34(7): 733–753.



Example : a soil layer of 5 m deep lying on a rock bed



Load history: a step load

The first stage: from zero to 1 KPa in 0.1 s

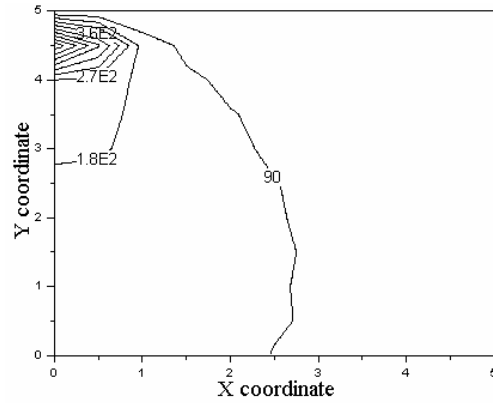
The second stage: remains constant

不可压缩条件:  $Q^* \rightarrow \infty$

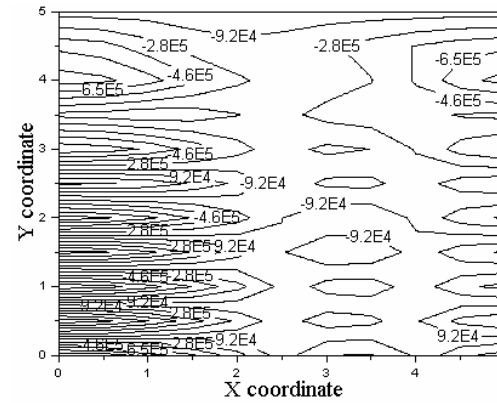
不排水边界条件:  $k = 0$

# 最大时间步长限制-I

压力分布  
at time t=1s

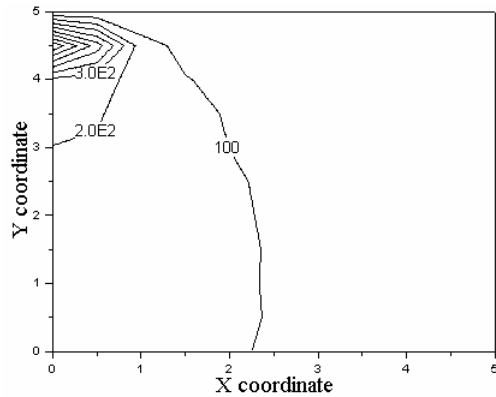


$$\Delta t = 10^{-4}$$

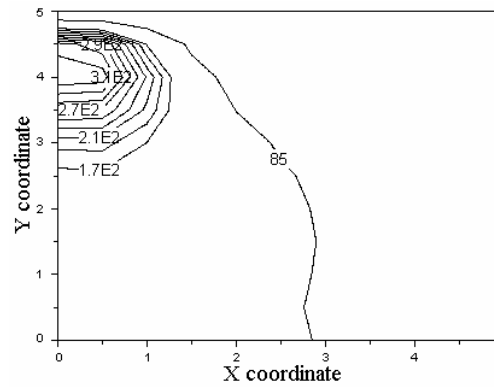


$$\Delta t = 10^{-3}$$

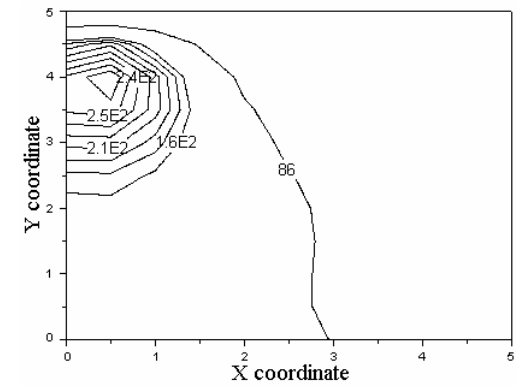
**FSA**  
(只允许小时间步长)



$$\Delta t = 10^{-3}$$



$$\Delta t = 2 \times 10^{-3}$$

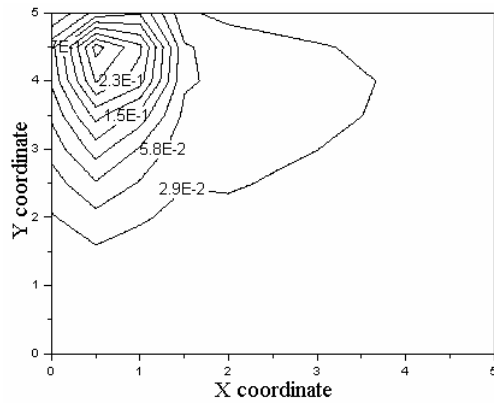


$$\Delta t = 5 \times 10^{-3}$$

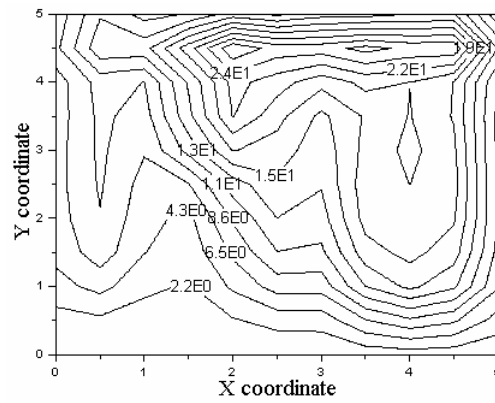
**PS-IFSA** (允许大时间步长)

# 最大时间步长限制-II

## 等价塑性应变分布 at time t=5s



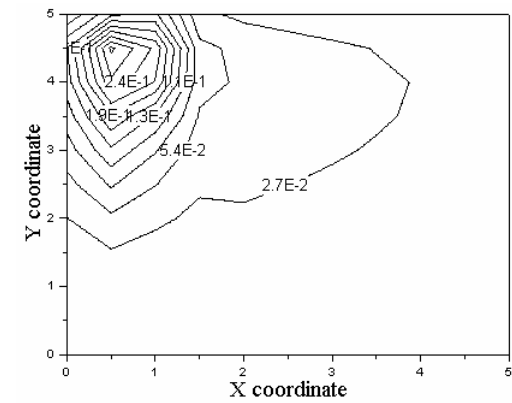
$$\Delta t = 10^{-4}$$



$$\Delta t = 10^{-3}$$

**FSA**

(只允许小时间步长)



$$\Delta t = 10^{-3}$$

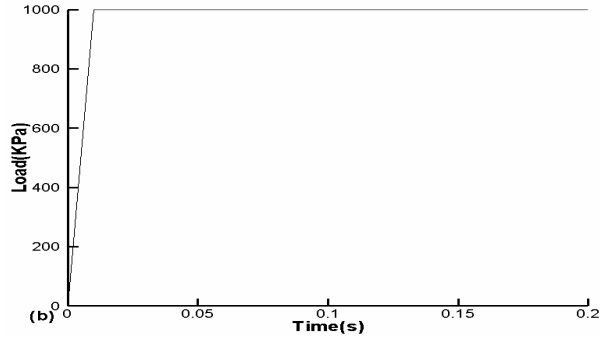
**PS-IFSA**

(允许大时间步长)

# 最小时间步长要求

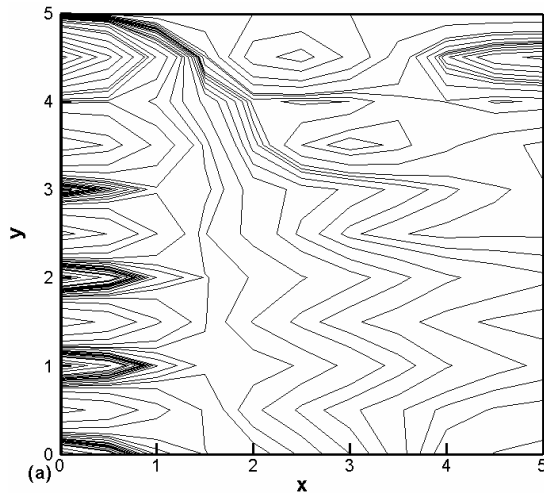
## 压力分布

at time  $t = 0.06$  s

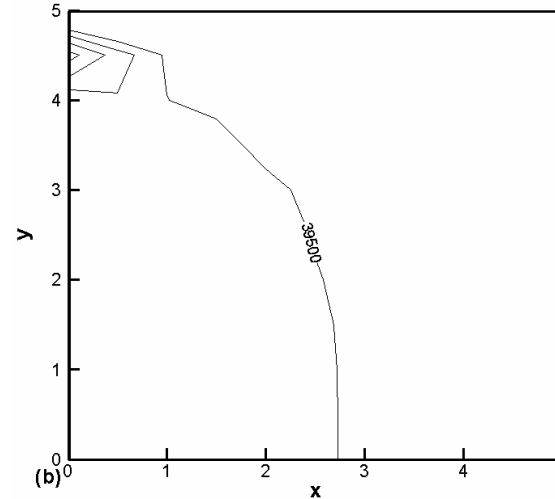


step loading with a ramp between  
 $t = 0$  and  $t = 0.01$  s

$\Delta t = 10^{-4}$  s (在极小时间步长下算法的可行性)



IFSA-N

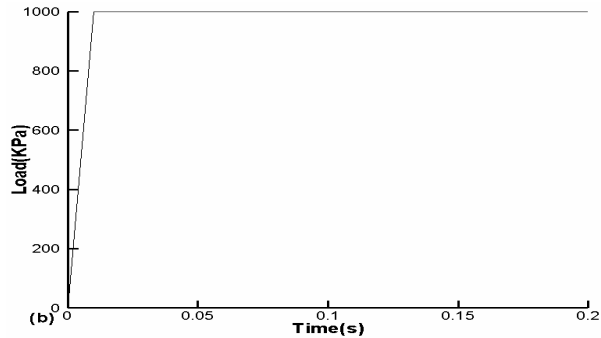


PS-IFSA-N

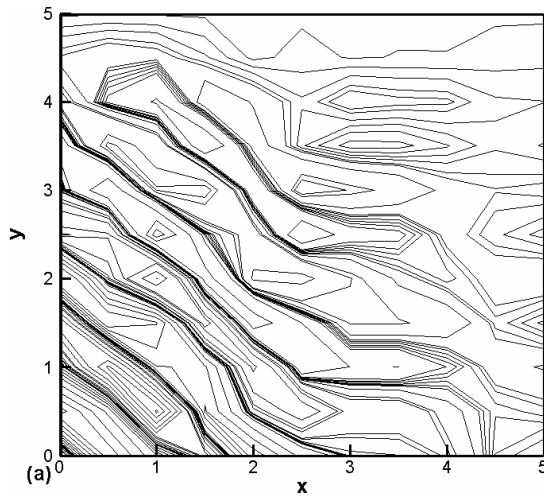
# 必须采用压力全量的分步算法-I

压力分布

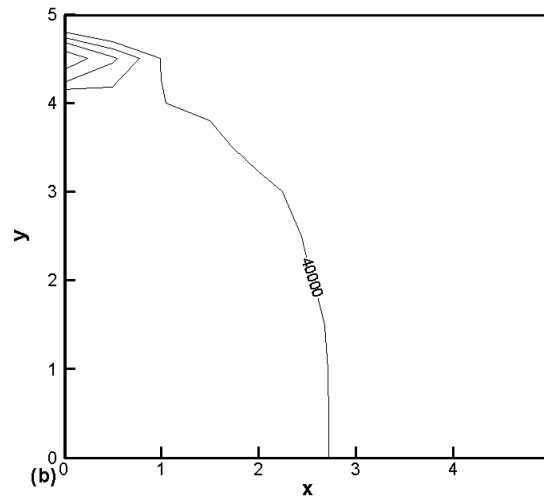
at time  $t = 0.06$  s



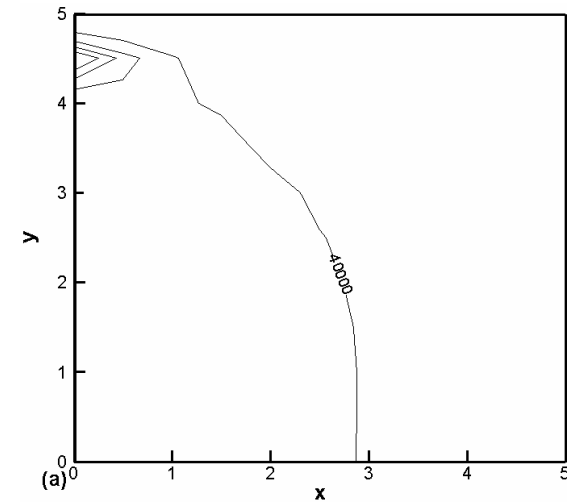
step loading with a ramp between  
 $t = 0$  and  $t = 0.01$  s



IFSA-I  
 $\Delta t = 10^{-3}$  s



PS-IFSA-I  
 $\Delta t = 10^{-3}$  s



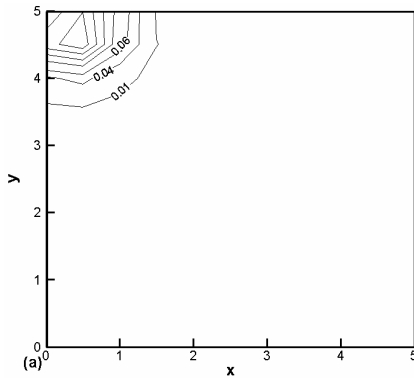
PS-IFSA-I  
 $\Delta t = 10^{-4}$  s

# 必须采用压力全量的分步算法-II

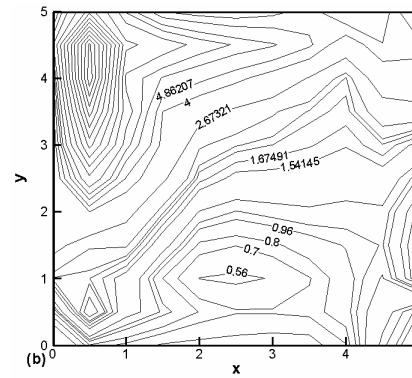
等价塑性应变分布

at time  $t = 0.06$  s

$$\Delta t = 10^{-3} \text{ s}$$

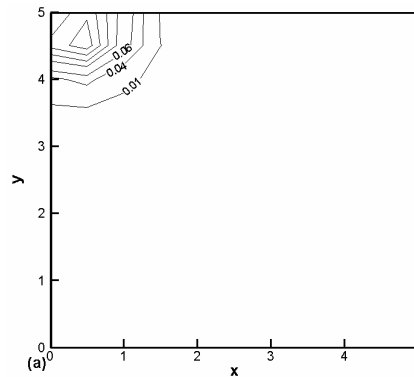


$t = 0.02$  s

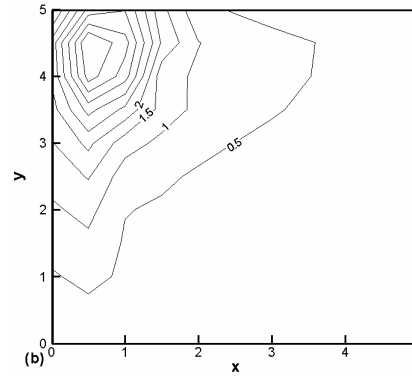


$t = 0.1$  s

IFSA-I



$t = 0.02$  s



$t = 0.1$  s

PS-IFSA-I

# 饱和多孔介质的梯度塑性和线性互补模型及有限元-无网格耦合方法

## 应变局部化问题正则化机制

### ➤ 梯度塑性连续体理论 (非局部连续体理论)

$$\sigma_y = \sigma_{y0} + h_p (\bar{\epsilon}^p) \bar{\epsilon}^p - \underline{C} \nabla^2 \bar{\epsilon}^p \quad C: \text{内尺度}$$

- Xikui Li, S Cescotto.

[Finite element method for gradient plasticity at large strains](#)  
**Int. J. for Numer. Methods in Eng., 1996, Vol.39, 619-633.**

- Xikui Li, S Cescotto.

[A mixed element method in gradient plasticity for pressure dependent materials and modelling of strain localization,](#)  
**Comput Methods in Appl Mech and Eng, 1997, Vol.144, 287-305.**

### ➤ Cosserat连续体理论 (具高阶连续结构的非经典连续体) $(\mathbf{u} = [u_x \ u_y \ \underline{\omega_z}]^T)$

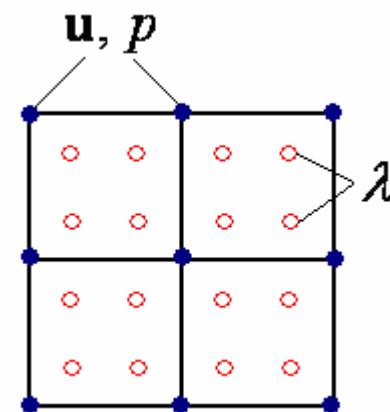
- Xikui Li, Hongxiang Tang

[A consistent return mapping algorithm for pressure-dependent elastoplastic Cosserat continua and modelling of strain localisation](#)  
**Computers and Structures, 83(1):1-10, 2005.**

## 梯度塑性连续体数值方法特点

等价塑性应变的Laplacian项  $\nabla^2 \bar{\varepsilon}^p$  要求:

- 节点基本未知量  $\mathbf{u}, p, \lambda$
- 塑性乘子  $\lambda$  保证至少具有  $C^1$  连续性的插值近似



### 有限元法

- ✓ 易于处理强制边界条件
- ✓ 系统刚度矩阵带宽小，计算效率高

$\mathbf{u}, p$ : 有限元插值;  
独立离散自由度定义在  
在有限元节点;

### 无网格法

- ✓ 形函数容易满足高阶连续性要求，无需增加额外的“节点”

$\lambda$ : 无网格插值;  
独立离散自由度定义  
在有限元网格的积分点;



## 所提出方法的其它特点

➤ 归结为线性互线性互补问题 (LCP) 标准型的梯度塑性模型特点:

(1) LCP标准型中同时包含了

● 平衡方程的弱形式 +

● 逐点满足(而不是积分意义下满足的)非局部本构方程与屈服条件

(2) 因而,对本模型所发展的一致性算法,使得离散化平衡方程与每个积分点的非局部本构方程和屈服准则在每次全局迭代中同时满足;

➤ 在保证二阶收敛率的同时无需形成非局部一致性切线刚度矩阵;

➤ 即使对于非关联塑性模型,所导出的为LCP求解的全局广义刚度阵仍保持为对称;

• Junbo Zhang, Xikui Li\*

A mixed finite element and mesh-free method using linear complementary theory for gradient plasticity in solids and saturated porous media

Computational Mechanics 47(2): 171-185, 2011.

# 间断 Galerkin 有限元法 (DGFEM)

饱和多孔介质动力控制方程的半离散系统的线性形式

$$\mathbf{M}\ddot{\mathbf{d}}(t) + \mathbf{C}\dot{\mathbf{d}}(t) + \mathbf{K}\mathbf{d}(t) = \mathbf{f}^e(t)$$

$$t \in I = (0, T)$$

$$\mathbf{d} = \begin{Bmatrix} \bar{\mathbf{u}} \\ \bar{\mathbf{U}} \end{Bmatrix}, \quad \dot{\mathbf{d}} = \begin{Bmatrix} \dot{\bar{\mathbf{u}}} \\ \dot{\bar{\mathbf{U}}} \end{Bmatrix}, \quad \ddot{\mathbf{d}} = \begin{Bmatrix} \ddot{\bar{\mathbf{u}}} \\ \ddot{\bar{\mathbf{U}}} \end{Bmatrix}, \quad \mathbf{f}^e = \begin{Bmatrix} \mathbf{f}_u^e \\ \mathbf{f}_U^e \end{Bmatrix}$$

$$\dot{\bar{\mathbf{U}}} = \dot{\bar{\mathbf{u}}} + \frac{\dot{\mathbf{w}}}{n} \quad \text{时间域能离散为 } 0 < t_1 < \cdots < t_n < t_{n+1} < \cdots < t_N = T$$

$$\mathbf{C} = \begin{bmatrix} \mathbf{C}_{11} & -\mathbf{C}_{12} \\ -\mathbf{C}_{12}^T & \mathbf{C}_{22} \end{bmatrix} \quad \begin{aligned} \mathbf{C}_{11} &= \int_{\Omega} N_k^u n^2 \delta_{ij} k_w^{-1} N_L^u d\Omega; \quad \mathbf{C}_{12} = \int_{\Omega} N_k^u n^2 \delta_{ij} k_w^{-1} N_L^U d\Omega \\ \mathbf{C}_{22} &= \int_{\Omega} N_k^U n^2 \delta_{ij} k_w^{-1} N_L^u d\Omega \end{aligned}$$

## ➤动力响应问题分类和传统有限元数值求解-问题的提出

### 动力响应问题分类

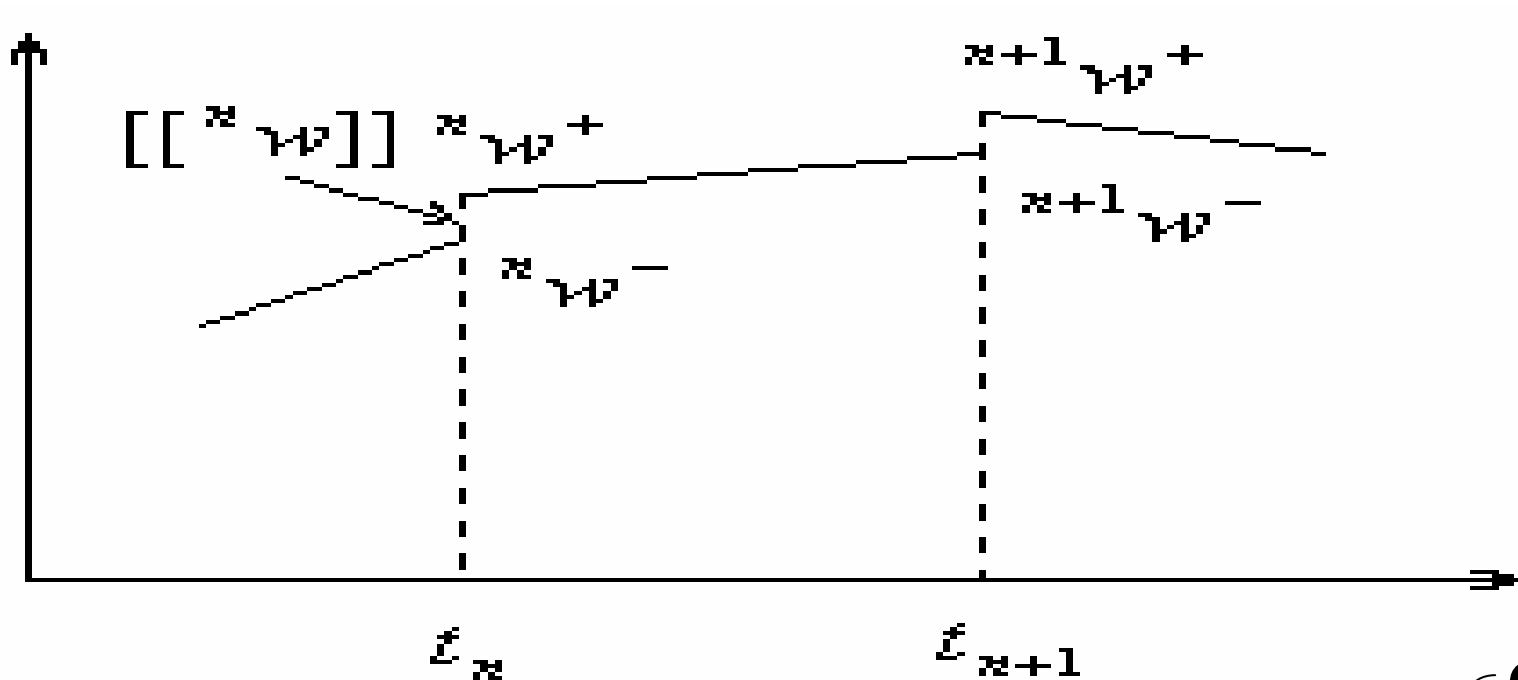
- 低频模式占主导的动力响应; (拟静力, 缓慢运动、如固结现象)
- 中频模式占主导的动力响应; (地震响应, 机械振动响应)
- 高频模式占主导的动力响应; (脉动、冲击响应, 爆炸荷载响应)

### 传统有限元求解过程(time-Continuous Galerkin FEM) (CGFEM)

- 动力控制方程在空间域利用有限元过程离散;
- 半离散化的动力控制方程在时间域利用有限差分过程离散  
(利用Newmark法等)
- 对模拟低频和中频模式占主导的动力响应, 通常能得到满意的数值求解结果;
- 而对模拟高频模式占主导的动力响应则得到很差的数值结果:
  - 不能捕捉住由于空间域中脉动波传播导致的间断解或高梯度解;
  - 不能滤去虚假的高阶模式效应和控制虚假的数值振荡现象。

## DGFEM基本特点-I

- 在空间域和时间域同时利用有限元过程离散;
- 在(空间)半离散控制方程中假定节点**基本变量向量** ( $\mathbf{d}$ )及其对**时间导数** ( $\mathbf{v}$ ) 在**时域中独立分片**多项式插值;
- 在离散的时域点, 允许 ( $\mathbf{d}$ )和( $\mathbf{v}$ )间断.



$$[[\mathbf{w}_n]] = \mathbf{w}(t_n^+) - \mathbf{w}(t_n^-) \quad \mathbf{w}(t_n^\pm) = \lim_{\varepsilon \rightarrow 0^\pm} \mathbf{w}(t_n + \varepsilon) \quad \mathbf{W} \longrightarrow \begin{cases} \mathbf{d}(t) \\ \mathbf{v}(t) \end{cases}$$

## DGFEM基本特点-II

$\mathbf{d}(t)$ 及其时间导数 $\mathbf{v}(t)$ 在时域中的分片独立插值,  
半离散动力控制方程可写成带约束条件的两变量形式

$$\mathbf{M}\dot{\mathbf{v}} + \mathbf{C}\mathbf{v} + \mathbf{K}\mathbf{d} = \mathbf{f}^e \quad \dot{\mathbf{d}} - \mathbf{v} = \mathbf{0}$$

双变量半离散动力控制方程和约束条件在子时域  $I_n = (t_n, t_{n+1})$   
中的弱形式, 连同双变量的间断条件可表示为

$$\int_{I_n} \delta \mathbf{v}^T (\mathbf{M}\dot{\mathbf{v}} + \mathbf{C}\mathbf{v} + \mathbf{K}\mathbf{d} - \mathbf{f}^e) dt + \delta \mathbf{d}_n^T \mathbf{K}[[\mathbf{d}_n]] + \int_{I_n} \delta \mathbf{d}^T \mathbf{K}(\dot{\mathbf{d}} - \mathbf{v}) dt + \delta \mathbf{v}_n^T \mathbf{M}[[\mathbf{v}_n]] = 0$$

已有DGFEM的缺点:  $\mathbf{d}_n^+ \neq \mathbf{d}_n^-$   $\mathbf{v}_n^+ \neq \mathbf{v}_n^-$

问题:

对一任意时间步  $I_n = (t_n, t_{n+1})$ , 对每一积分点  $\sigma(\mathbf{d}_n^+)$  必须要被重新计算。即: 在一个增量时间步的两端, 每一积分点的  $\sigma(\mathbf{d}_n^+)$  和  $\sigma(\mathbf{d}_{n+1}^-)$  都要同时进行计算。意味着对于非线性问题, 需要化两倍的计算工作量在耗时的满足积分点处非线性本构关系迭代。

## 本工作 DGFEM 基本特点

假定半离散控制方程中节点基本变量向量  $\mathbf{d}(t)$  在时域中分片三阶多项式 (Hermite) 形函数 (P3) 插值, 即  $\forall t \in I_n = (t_n, t_{n+1})$

$$\mathbf{d}(t) = \mathbf{d}_n^+ N_1(t) + \mathbf{d}_{n+1}^- N_2(t) + \mathbf{v}_n^+ M_1(t) + \mathbf{v}_{n+1}^- M_2(t)$$

而其对时间导数  $\mathbf{v}(t)$  在时域中分片一阶多项式 (线性) 形函数 (P1) 插值

$$\mathbf{v}(t) = \mathbf{v}_n^+ \lambda_1(t) + \mathbf{v}_{n+1}^- \lambda_2(t)$$

即  $\mathbf{d}(t)$  和  $\mathbf{v}(t)$  在时域中的分片 P3-P1 形函数插值。代入弱形式

$$\delta \mathbf{d}_n^T \mathbf{K}[[\mathbf{d}_n]] + \int_{I_n} \delta \mathbf{d}^T \mathbf{K}(\dot{\mathbf{d}} - \mathbf{v}) dt + \int_{I_n} \delta \mathbf{v}^T (\mathbf{M}\dot{\mathbf{v}} + \mathbf{C}\mathbf{v} + \mathbf{K}\mathbf{d} - \mathbf{f}^e) dt + \delta \mathbf{v}_n^T \mathbf{M}[[\mathbf{v}_n]] = 0$$

虽然仍假定允许  $\mathbf{d}(t)$  和  $\mathbf{v}(t)$  在  $t_n, t_{n+1}$  处间断, 但弱形式给出

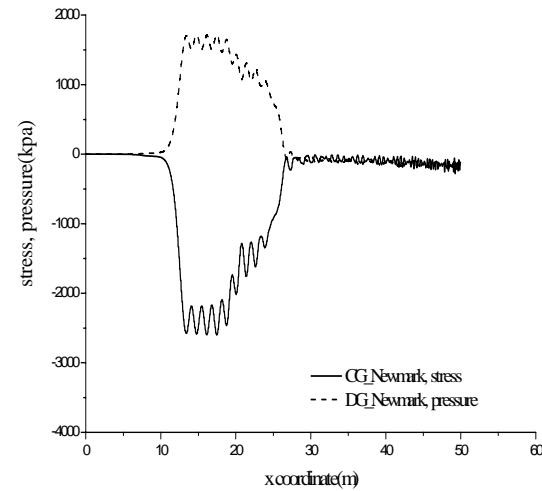
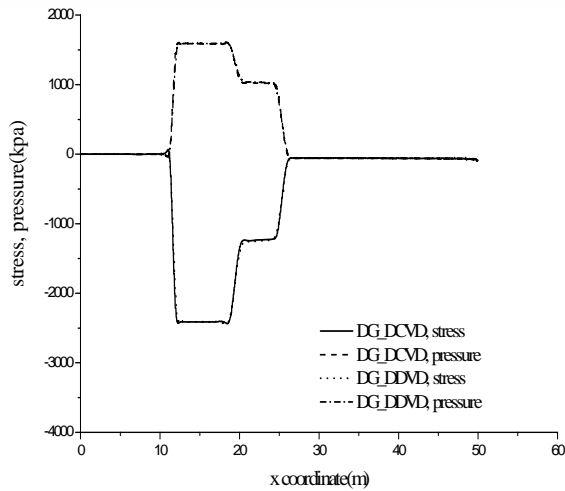
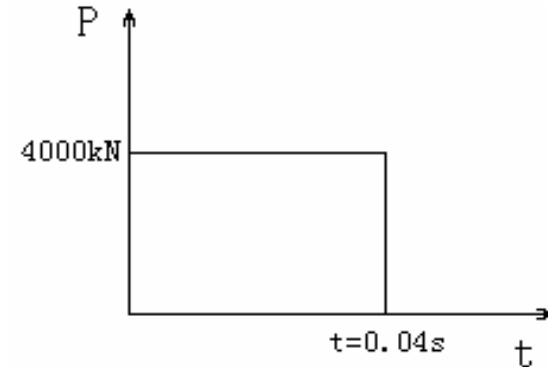
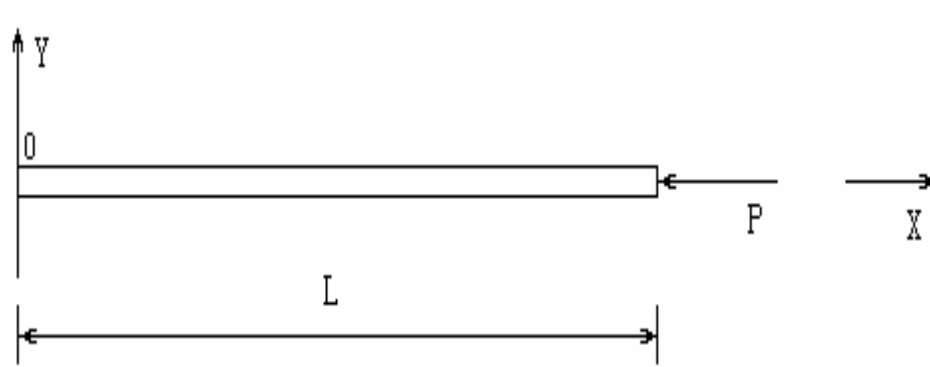
$$\mathbf{d}_n^+ = \mathbf{d}_n^- \quad \mathbf{v}_n^+ \neq \mathbf{v}_n^-$$

Xikui Li, Dongmei Yao, R.W.Lewis. A discontinuous Galerkin finite element method for dynamic and wave propagation problems in non-linear solids and saturated porous media, *Int. J. for Numerical Methods in Eng.* 2003, **57**(12): 1775:1800.

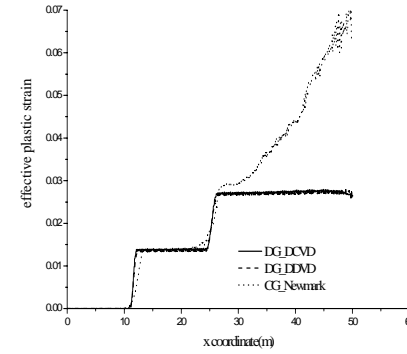
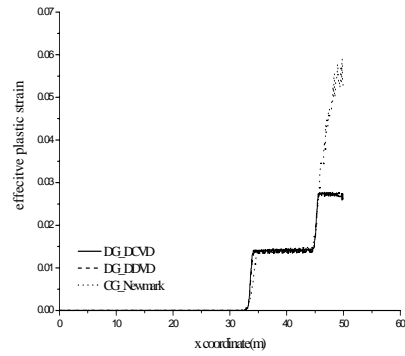
# 算例 1: 饱和介质一维压力波问题

弹塑性情况, Drucker-Prager准则

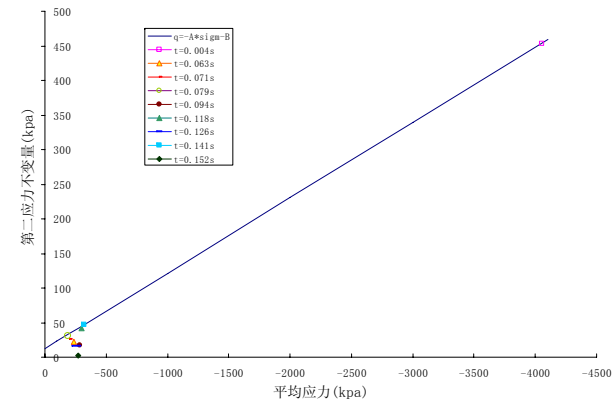
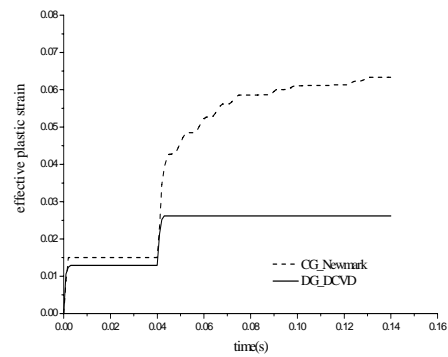
排水条件:  $U_x \neq u_x, w_x \neq 0$  (轴向)



排水条件下饱和多孔介质柱中弹塑性压力波的传播. 有效应力和孔隙压力波形的比较(t=0.14s). 本文DG\_DCVD方法与已有DG\_DDVD方法; CG\_Newmark方法.



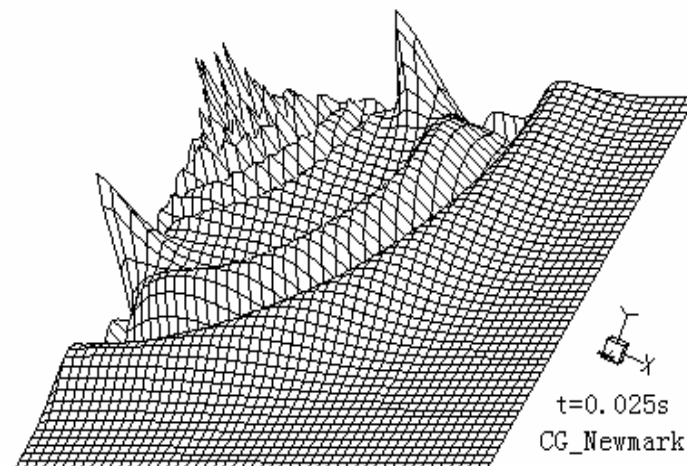
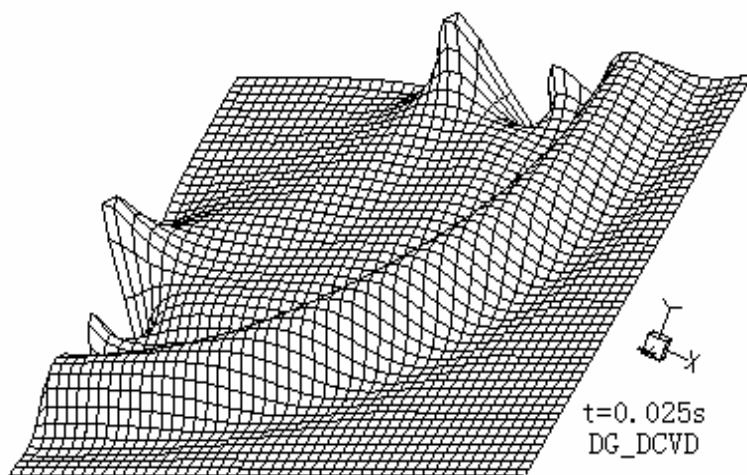
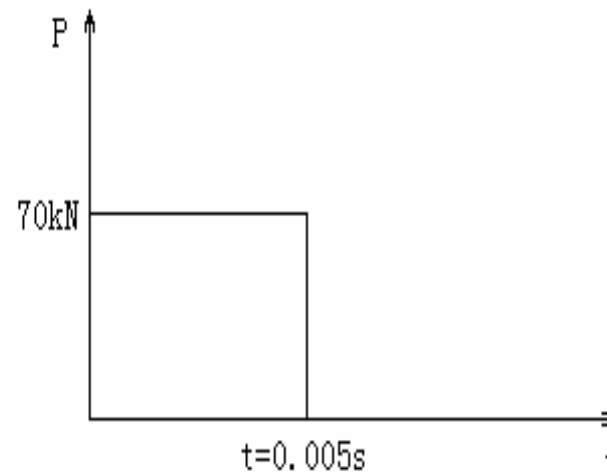
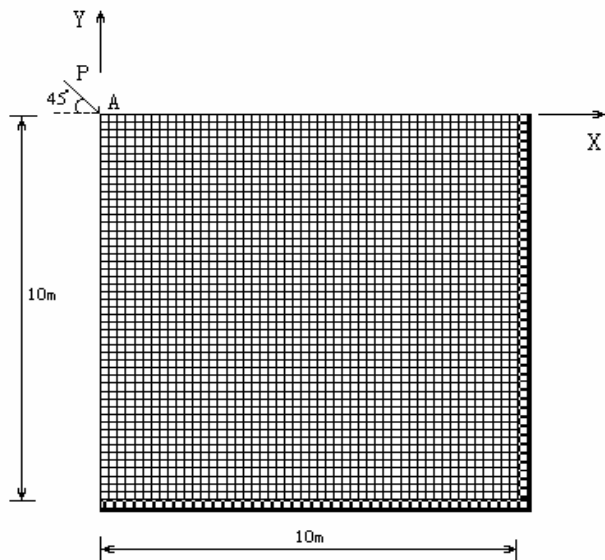
排水条件下饱和多孔介质柱中等效塑性应变分布. 本文 DG\_DCVD方法、已有 DG\_DDVD方法和CG\_Newmark方法的结果比较. (a)  $t=0.06s$ 时刻; (b)  $t=0.14s$ 时刻.



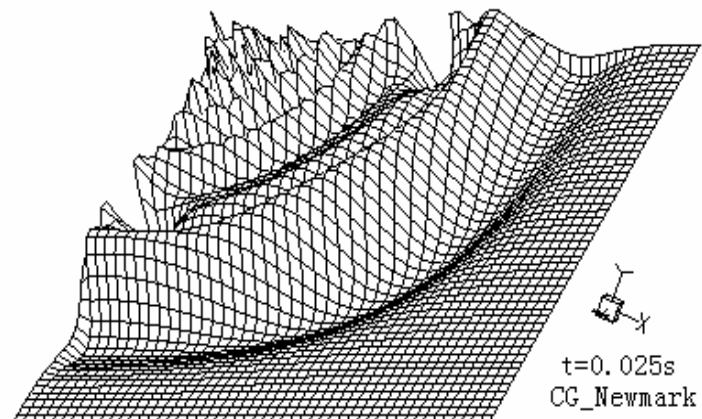
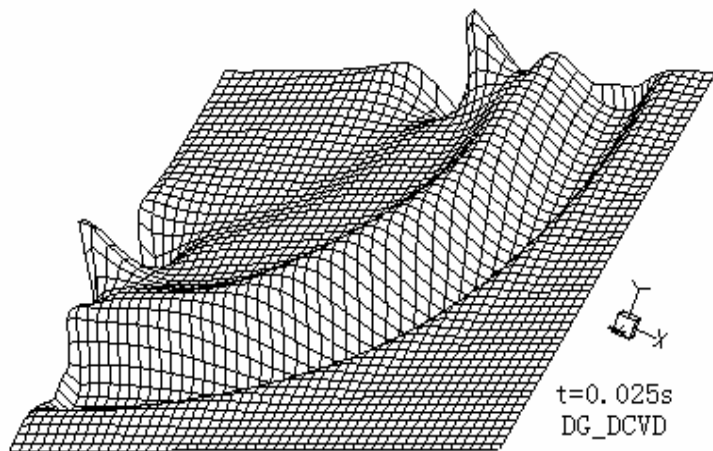
饱和多孔介质柱自由端处等效塑性应变  
随时间的变化. 本文DG\_DCVD方法与  
CG\_Newmark方法的结果比较



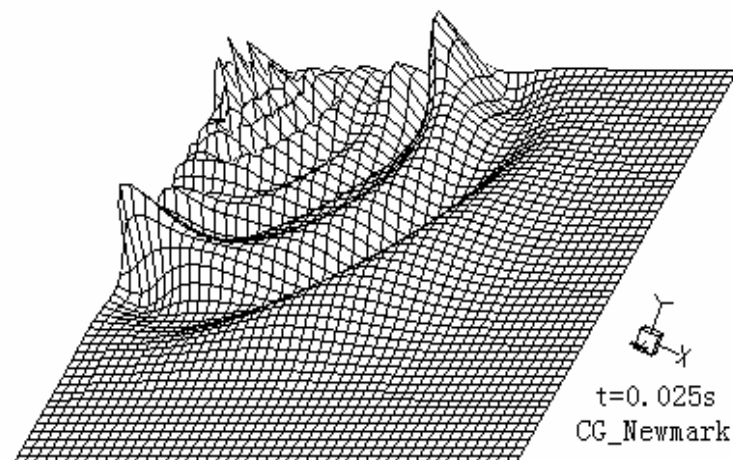
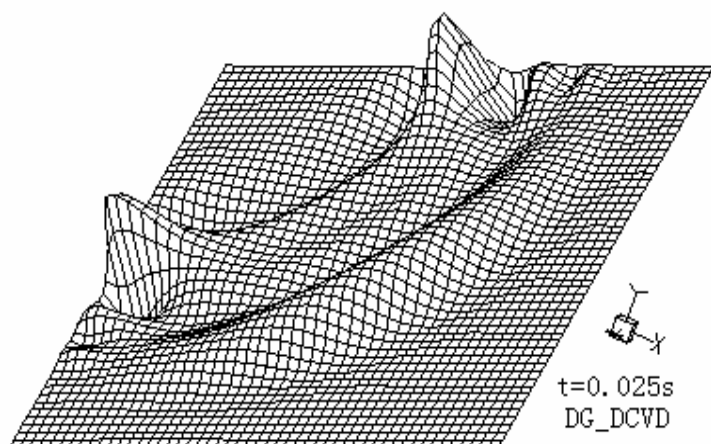
## 算例 2: 饱和弹性多孔介质中的二维波传播



内水压力分布( $t=0.025s$ ) (a) 本文DG\_DCVD方法; (b) CG\_Newmark方法.



球应力分布( $t=0.025s$ ) (a) 本文DG\_DCVD方法; (b) CG\_Newmark方法



剪应力分布( $t=0.025s$ ) (a) 本文DG\_DCVD方法; (b) CG\_Newmark方法

# 饱和/非饱和多孔介质中热-渗流(湿、气)-力学耦合分析的混合有限元方法

## 问题的提出

- (1) 模拟应变局部化现象的两方面需要:
  - 在经典连续体中引入某种类型的正则化机制(梯度塑性模型, Cosserat连续体模型)以保持问题的适定性;
  - 单元的内禀性态(排除剪切、不可压缩自锁, 允许采用稀疏和歪扭网格等)应有能力再现承载能力下降的软化特征。
- (2) 不可压缩、不排水饱和多孔介质空间离散导致可约化u-p混合公式, **LBB条件**排除了所期待的等低阶普通单元的应用。
- (3) 从计算效率角度要求发展**低阶高精度单元**(特别对于多场耦合问题)

- 
- 基于固体的“稳定一点积分超收敛元”(Belytschko)
  - 基于 Xikui Li, S. Cescotto. Finite element method for gradient plasticity at large strains, Int. J. for Numer. Methods in Eng., 1996, Vol.39, 619-633.

## 热耦合非饱和 (含液体, 干空气, 蒸气) 多孔介质中 每个耦合相的独立变量

### Three independent variables for each coupled phase (THM)

- The solid phase:  $\mathbf{u}, \varepsilon, \sigma^*$
- The pore water phase:  $p_w, \mathbf{g}_w, \dot{\mathbf{w}}_w$
- The pore gas phase:  $p_a, \mathbf{g}_a, \dot{\mathbf{w}}_a$
- The temperature field:  $T, \mathbf{g}_T, \mathbf{q}_T$

$\mathbf{g}_w, \mathbf{g}_a, \mathbf{g}_T$  - gradients of  $p_w, p_a, T$  in space

$\dot{\mathbf{w}}_w, \dot{\mathbf{w}}_a, \mathbf{q}_T$  - Darcy's velocities and heat flux

## The weak form of three-field variational approach

$$\delta\Pi = \delta\Pi_s + \delta\Pi_w + \delta\Pi_a + \delta\Pi_T$$

$$\begin{aligned} \delta\Pi_s = & \int_{A_e} [\delta\boldsymbol{\varepsilon}^T (\mathbf{D}\boldsymbol{\varepsilon} + \mathbf{m}(\pi p_c - K(\beta_s \Delta T + \beta_c c_p))) - \boldsymbol{\sigma}^*] \\ & + \delta\boldsymbol{\sigma}^{*T} (\boldsymbol{\varepsilon} - \nabla\mathbf{u}) + \delta(\nabla\mathbf{u})^T (\boldsymbol{\sigma}^* - \mathbf{m}p_a) - \delta\mathbf{u}^T \rho(\mathbf{b} - \ddot{\mathbf{u}})] dA_e \end{aligned}$$

$$\begin{aligned} \delta\Pi_w = & \int_{A_e} \delta p_w \left( \frac{\partial(nS_w \rho_w)}{\partial t} + \nabla \cdot (nS_w \rho_w \dot{\mathbf{u}}) + \nabla \cdot (\rho_w \dot{\mathbf{w}}_w) \right) dA_e \\ & + \int_{A_e} [\delta\dot{\mathbf{w}}_w^T (\mathbf{g}_w - \nabla p_w) + \delta\mathbf{g}_w^T (\dot{\mathbf{w}}_w + \mathbf{k}_w (\mathbf{g}_w - \rho_w (\mathbf{b} - \ddot{\mathbf{u}})))] dA_e \end{aligned}$$

基于胡海昌—Washizu三变量变分原理

## The weak form of three-field variational approach

$$\begin{aligned}
 \delta\Pi_a &= \int_{A_e} \delta p_a \left( \frac{\partial(nS_a \rho_a)}{\partial t} + \nabla \cdot (nS_a \rho_a \dot{\mathbf{u}}) + \nabla \cdot (\rho_a \dot{\mathbf{w}}_a) \right) dA_e \\
 &+ \int_{A_e} \left[ \delta \dot{\mathbf{w}}_a^T (\mathbf{g}_a - \nabla p_a) + \delta \mathbf{g}_a^T (\dot{\mathbf{w}}_a + \mathbf{k}_a (\mathbf{g}_a - \rho_a (\mathbf{b} - \dot{\mathbf{u}}))) \right] dA_e \\
 \delta\Pi_T &= \int_{A_e} \left[ \delta \mathbf{g}_T^T \left( \mathbf{q}_T + \left( \mathbf{k}_T \mathbf{g}_T - \left( \phi_w \frac{\dot{\mathbf{w}}_w}{nS_w} + \phi_a \frac{\dot{\mathbf{w}}_a}{nS_a} \right) \right) \right) \right. \\
 &\quad \left. \delta \mathbf{q}_T^T (\mathbf{g}_T - \nabla T) \right] dA_e \\
 &+ \int_{A_e} \delta T \left( \nabla \cdot \left( \phi_w \left( \dot{\mathbf{u}} + \frac{\dot{\mathbf{w}}_w}{nS_w} \right) \right) + \frac{\partial \phi_a}{\partial t} + \nabla \cdot \left( \phi_a \left( \dot{\mathbf{u}} + \frac{\dot{\mathbf{w}}_a}{nS_a} \right) \right) \right) \\
 &\quad \left( + \frac{\partial \phi_w}{\partial t} + \frac{\partial \phi_s}{\partial t} + \nabla \cdot (\phi_s \dot{\mathbf{u}}) - \nabla \cdot (\mathbf{k}_T \mathbf{g}_T) \right) dA_e
 \end{aligned}$$

$\delta\Pi_s, \delta\Pi_w, \delta\Pi_a, \delta\Pi_T$  分别由三个弱形式项组成, 即

(a) 守恒方程弱形式, 即

各相混合体动量守恒, 孔隙水质量守恒, 孔隙气质量守恒, 能量守恒;

(b) 各基本变量及其空间梯度之间关系弱形式;

(c) 本构定律弱形式, 即

应力应变关系, 孔隙水与气的Darcy定律, 热传导与热对流律.

• 非线性混合有限元公式和一致性算法

材料非线性: 非光滑多重(热-湿-气耦合)弹塑性屈服面下的率本构方程积分和一致性切线模量张量;

几何非线性: 共旋公式途径

• Xikui Li, Zejia Liu, R.W. Lewis, Kiichi Suzuki. **Mixed finite element method for saturated poro-elasto-plastic media at large strains,**

**Int. J. Numer. Methods Eng. 2003, 57(6): 875-898.**

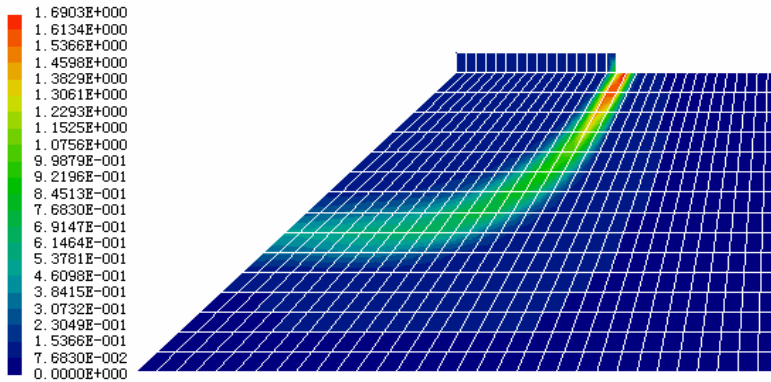
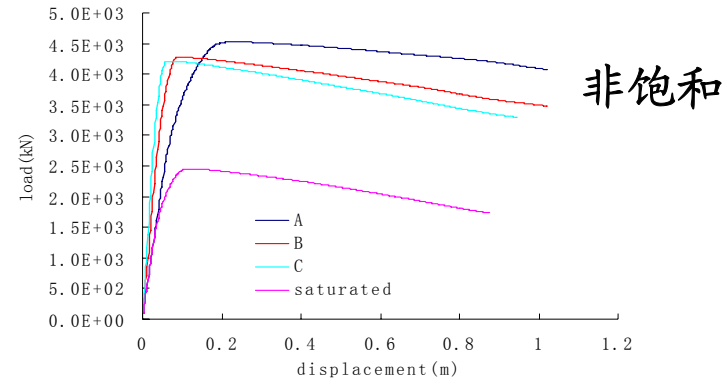
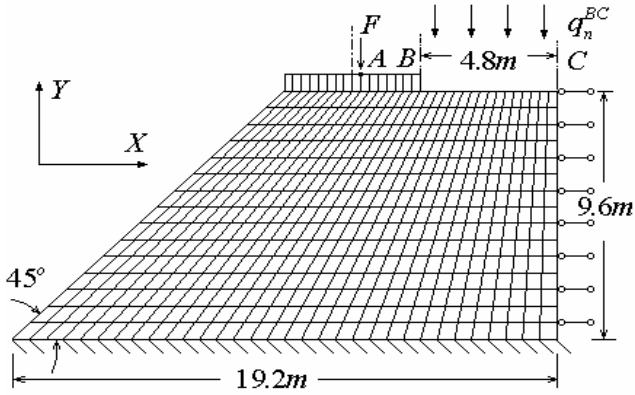
• Xikui Li, Zejia Liu, R.W. Lewis. **Mixed finite element method for coupled thermo-hydro-mechanical process in poro-elasto-plastic media at large strains**

**Int. J. for Nume. Methods in Eng. 2005, 64(5): 667-708.**

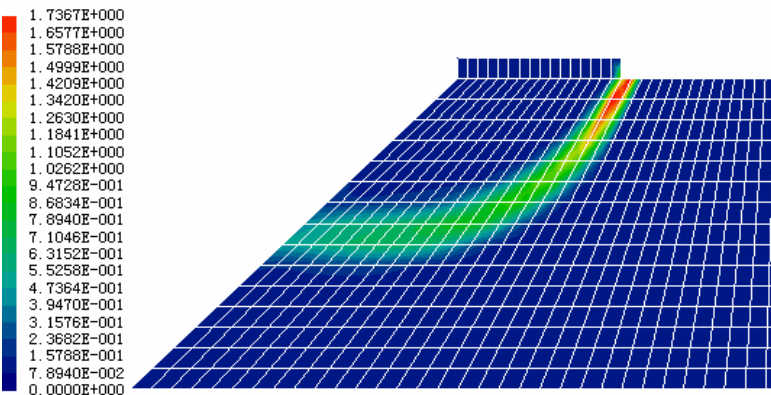
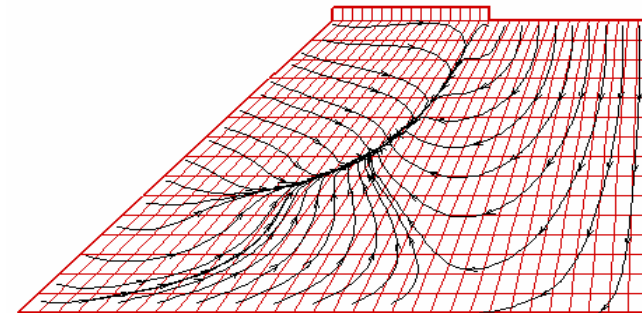
# 边坡稳定性：应变局部化模拟

**A:**  $\dot{u}_y^A = -2 \times 10^{-3} \text{ m/s}$     **B:**  $\dot{u}_y^A = -2 \times 10^{-3} \text{ m/s}$      $q_n^{BC} = 0.75 \text{ m/s}$

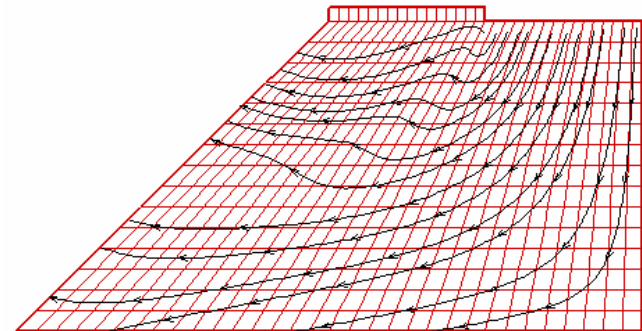
**C:**  $\dot{u}_y^A = -2 \times 10^{-3} \text{ m/s}$      $q_n^{BC} = 1.15 \text{ m/s}$     Saturated: -A



Case A



Case C





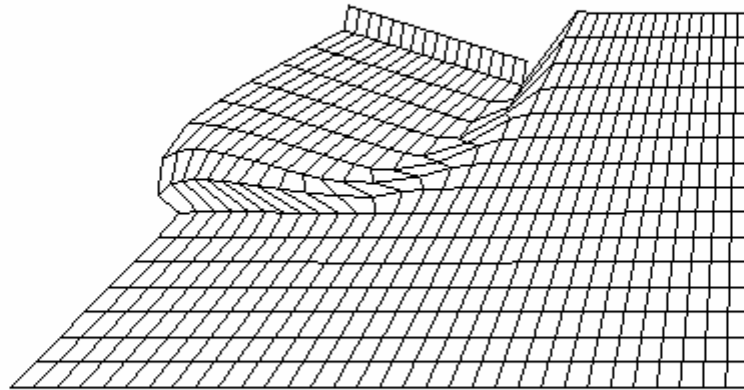


Fig. 3. Deformed configuration of the slope with 480 mixed element mesh in the high loading rate at the end of the load history

# 高温下混凝土化学 - 热 - 湿 - 气 - 力学 (CTHM) 过程全耦合分级数学模型、本构模拟与数值方法

高性能混凝土 (HPC) 由于它的高强度和耐久性广泛应用于高层建筑和桥梁、隧道、海洋结构、核工程等基础设施。



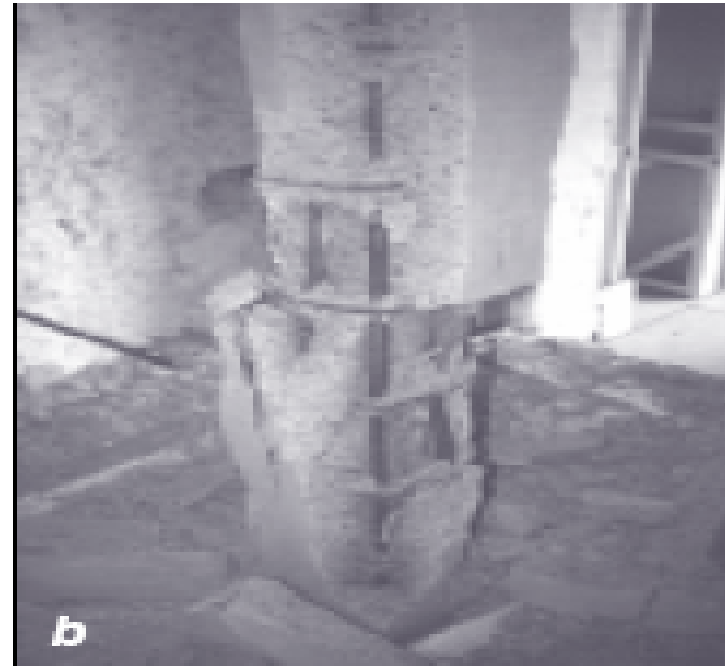
但与普通混凝土 (NSC) 相比，它在高温下的材料强度丧失率和材料损伤率要高得多。

更严重的在火灾高速率高温下，易于发生危害更大甚至导致结构崩塌的爆裂破坏现象。

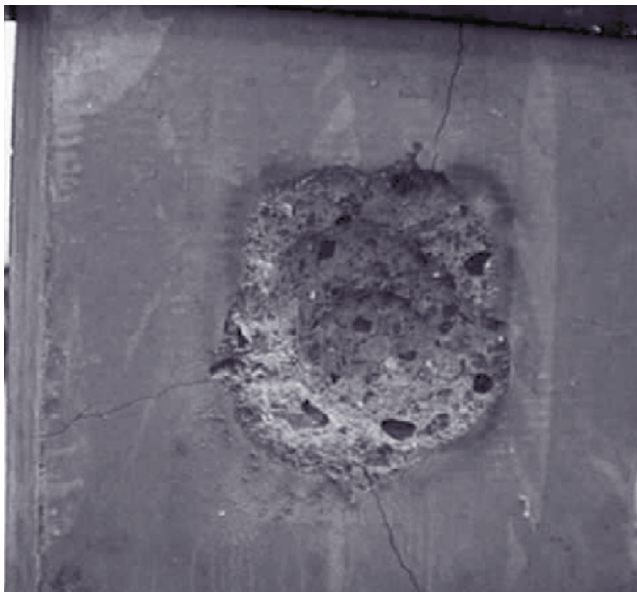




a - View of NSC after fire-resistance tests



b - View of HPC after fire-resistance tests



## 爆裂破坏现象

Spalling of fire-exposed concrete in Hertz's test, 2003.

# 研究方案

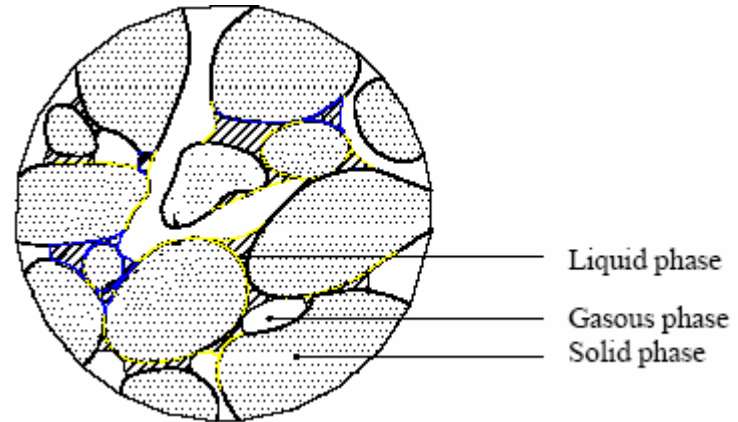
- 建立HPC作为非饱和变形多孔多相介质在高温高速率热载作用下的热-湿-气-化学-力学全耦合数学模型。
- 发展相应的非线性有限元数值方法。
- 建立模拟热-湿-气-化学-力学耦合环境下HPC中非线性材料行为的本构模型，特别是模拟破坏过程的化学塑性-损伤耦合本构模型。研究高温下HPC中束缚水脱水、化学成份脱盐（相变）导致的化学-力学耦合、化学塑性-损伤耦合软化效应。
- 数值模拟HPC构件和结构中的热-湿-气-化学-力学响应和因高孔隙压力和约束膨胀引起的热损伤和热脱粘裂纹的发展和热爆裂现象。

# 耦合过程的数学模型- 数学(定量)描述

## 混凝土的**不混溶-混溶**两级模型

### 主级模型:

- 模型化为变形非饱和多孔连续介质;
- 由**不混溶三相**组成:  
混合固相; 混合液相; 混合气相;



### 次级模型:

the solid skeleton  
the matrix components  
chemically bound water } Solid mixtures (由**混溶三相**组成)

the matrix components  
(dissolved from the solid phase)  
capillary water (free water)  
physically adsorbed water } Liquid mixtures  
(由**混溶三相**组成)

dry air  
vapor } Gas mixtures (由**混溶两相**组成)

# 耦合过程的数学模型- 数学(定量)描述

## 相变过程

- **Dehydration:** (脱水)

solid matrix + energy  $\Rightarrow$  chemically adsorbed water

(固相  $\longrightarrow$  液相)

- **Desalination:** (脱盐)

solid matrix + energy  $\Rightarrow$  matrix components

(固相  $\longrightarrow$  液相)

- **Evaporation:** (蒸发)

capillary water + energy  $\Rightarrow$  water vapour

(液相  $\longrightarrow$  气相)

Xikui Li, Rongtao Li, B.A. Schrefler A coupled chemo-thermo-hygro-mechanical model of concrete at high temperature and failure Analysis,  
[Int. J. for Numerical and Analytical Methods in Geomechanics, 2006, 30\(7\): 635-681](#)

模型化为化学反应多孔连续介质、计及相变过程的  
混凝土的各相质量守恒方程

$$\frac{D\rho^s}{Dt} + \rho^s (\nabla \cdot \mathbf{v}) = -\dot{m}_{\text{dehy}} - \dot{m}_{\text{des}}$$

$$\frac{D\rho^w}{Dt} + \rho^w (\nabla \cdot \mathbf{v}) + \nabla \cdot (\rho^w \mathbf{v}_{ls}) = \dot{m}_{\text{dehy}} - \dot{m}_{\text{vap}}$$

$$\frac{D\rho^p}{Dt} + \rho^p (\nabla \cdot \mathbf{v}) + \nabla \cdot (\rho^p \mathbf{v}_{ls} + \mathbf{J}_l^p) = \dot{m}_{\text{des}}$$

$$\frac{D\rho^a}{Dt} + \rho^a (\nabla \cdot \mathbf{v}) + \nabla \cdot (\rho^a \mathbf{v}_{gs} + \mathbf{J}_g^a) = 0$$

$$\frac{D\rho^v}{Dt} + \rho^v (\nabla \cdot \mathbf{v}) + \nabla \cdot (\rho^v \mathbf{v}_{gs} + \mathbf{J}_g^v) = \dot{m}_{\text{vap}}$$

$\dot{m}_{\text{dehy}}$  - the rate of the lost mass of the solid phase due to the dehydration

$\dot{m}_{\text{des}}$  - the rate of the lost mass of the solid phase due to the desalination

$\dot{m}_{\text{vap}}$  - the rate of the lost mass of the liquid phase due to the evaporation

假定孔隙材料各相在任何局部点处于热力学平衡状态，任何点处各相具有相同温度。因而，对孔隙材料可作为一个混合体写出能量(焓)守恒方程

$$\begin{aligned} (\rho C_p)_{eff} \frac{\partial T}{\partial t} + [C_{pl} \rho_l \mathbf{v}_{ls} + C_{pg} \rho_g \mathbf{v}_{gs}] \cdot \nabla T - \nabla \cdot (\lambda_{eff} \nabla T) \\ = -\Delta h_{vap} \dot{m}_{vap} - \Delta h_{dehy} \dot{m}_{dehy} - \Delta h_{des} \dot{m}_{des} \end{aligned}$$

$$(\rho C_p)_{eff} = \rho^s C_{ps} + \rho^l C_{pl} + \rho^a C_{pa} + \rho^v C_{pv}$$

$C_{ps}$   $C_{pl}$   $C_{pa}$   $C_{pv}$  - 固相、液相、干空气和水蒸汽的等压热容

$\Delta h_{vap}$   $\Delta h_{dehy}$   $\Delta h_{des}$  - 水的蒸发焓、脱水焓和基质溶解焓  
(相变)



## 多孔多相混合介质的总体动量守恒方程

$$\nabla \cdot \boldsymbol{\sigma} + \rho \mathbf{g} = 0$$

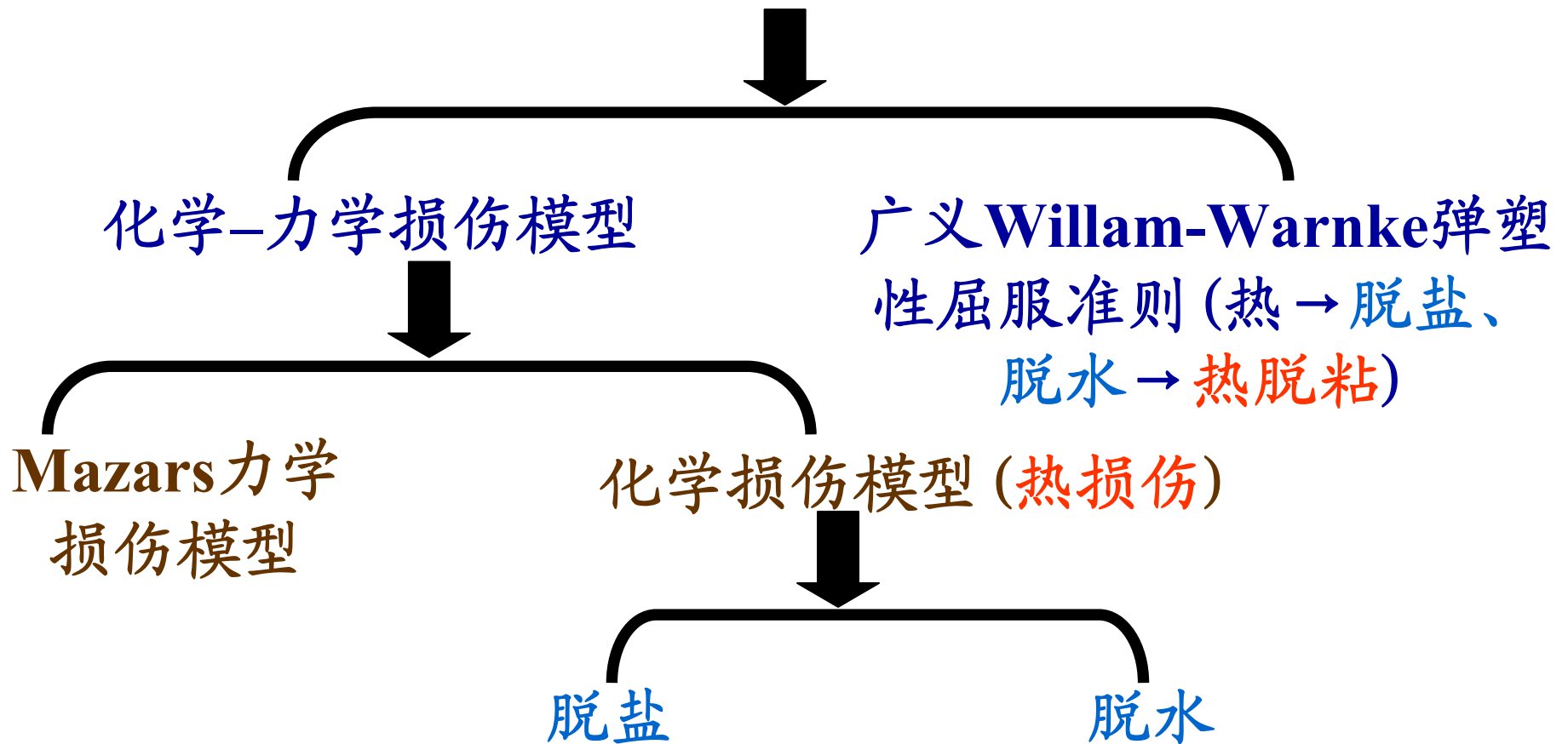
➤  $\boldsymbol{\sigma} = \boldsymbol{\sigma}'' - \alpha p_s \mathbf{I}$

➤ Effective stress tensor :  $\boldsymbol{\sigma}''$

➤ Biot constant :  $\alpha = 1 - \frac{K_T}{K_s} = 1 - \frac{\delta_{ij} D_{ijkl}^{ep} \delta_{kl}}{3K_s}$

- the bulk moduli of the solid material:  $K_s$
- the bulk moduli of the solid skeleton:  $K_T$
- 4<sup>th</sup> order elasto-plastic constitutive tensor:  $D_{ijkl}^{ep}$

# 高温下化学塑性-损伤耦合破坏本构模型



$$d = 1 - (1 - d_m)(1 - d_c) = 1 - (1 - d_m)(1 - d_s)(1 - d_h)$$

Rongtao Li, Xikui Li\*. A coupled chemo-elastoplastic-damage constitutive Model for plain concrete subjected to high temperature, *I J Damage Mechanics*, 19(8): 971-1000, 2010.

# 有限元离散与求解过程

## Choice of Primary Variables

- Displacements  $\underline{\mathbf{u}}$
- Gas Pressure  $P_g$
- Capillary Pressure  $P_c$
- Temperature  $T$
- Mass of Dissolved  
Matrix Component  $c^p$

## The natural (Neumann's) and the mixed (Cauchy's) boundary conditions

$$\mathbf{B}_1 = \boldsymbol{\sigma} \cdot \mathbf{n} = \bar{\mathbf{t}} \quad \text{on } \Gamma_t(\Gamma_1)$$

$$B_2 = (\rho^a \mathbf{v}_{gs} + \mathbf{J}_g^a) \cdot \mathbf{n} - q_a = 0 \quad \text{on } \Gamma_g(\Gamma_2)$$

$$B_3 = (\rho^w \mathbf{v}_{ls} + \rho^v \mathbf{v}_{gs} + \mathbf{J}_g^v) \cdot \mathbf{n} - [q_v + q_w + \beta_c (\rho_v - \rho_{v\infty})] = 0 \quad \text{on } \Gamma_c(\Gamma_3)$$

$$B_4 = (\rho^w \mathbf{v}_{ls} \Delta h_{vap} + \lambda_{eff} \nabla T) \cdot \mathbf{n} + [q_T + \alpha_c (T - T_\infty) + e \sigma_0 (T^4 - T_\infty^4)] = 0 \quad \text{on } \Gamma_T(\Gamma_4)$$

$$B_5 = (-\mathbf{D}_h \nabla c^p) \cdot \mathbf{n} - q_p = 0 \quad \text{on } \Gamma_p(\Gamma_5)$$

➤ 相应于如下五个基本控制方程:

- 混合体动量守恒
- 干空气质量守恒
- 湿(水+蒸汽)份质量守恒
- 混合体能量守恒
- 溶质(盐份等)质量守恒

## 控制方程的弱形式及相应的自然和混合边界条件

$$\begin{aligned}\mathbf{F}_u &= \mathbf{F}_u^i - \mathbf{f}_u^e = \int_{\Omega} \mathbf{W}_u \cdot \mathbf{L}_1 d\Omega + \int_{\Gamma_1} \mathbf{W}_u \cdot \mathbf{B}_1 d\Gamma_1 = \mathbf{0} \\ \mathbf{F}_g &= \mathbf{F}_g^i - \mathbf{f}_g^e = \int_{\Omega} \mathbf{W}_g L_2 d\Omega + \int_{\Gamma_2} \mathbf{W}_g B_2 d\Gamma_2 = \mathbf{0} \\ \mathbf{F}_w &= \mathbf{F}_w^i - \mathbf{f}_w^e = \int_{\Omega} \mathbf{W}_w L_3 d\Omega + \int_{\Gamma_3} \mathbf{W}_w B_3 d\Gamma_3 = \mathbf{0} \\ \mathbf{F}_T &= \mathbf{F}_T^i - \mathbf{f}_T^e = \int_{\Omega} \mathbf{W}_T L_4 d\Omega + \int_{\Gamma_4} \mathbf{W}_T B_4 d\Gamma_4 = \mathbf{0} \\ \mathbf{F}_c &= \mathbf{F}_c^i - \mathbf{f}_c^e = \int_{\Omega} \mathbf{W}_c L_5 d\Omega + \int_{\Gamma_5} \mathbf{W}_c B_5 d\Gamma_5 = \mathbf{0}\end{aligned}$$

➤ “内力向量”  $\mathbf{F}_u^i, \mathbf{F}_g^i, \mathbf{F}_w^i, \mathbf{F}_T^i, \mathbf{F}_c^i$

➤ “外力向量”  $\mathbf{f}_u^e, \mathbf{f}_g^e, \mathbf{f}_w^e, \mathbf{f}_T^e, \mathbf{f}_c^e$

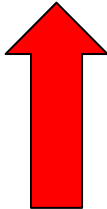
## Newton-Raphson procedure

$$\mathbf{F}^k(\mathbf{X}_{n+1}^k) = \left. \frac{\partial \mathbf{F}^k}{\partial \mathbf{X}} \right|_{\mathbf{X}_{n+1}^k} \Delta \mathbf{X}_{n+1}^k \quad \mathbf{X}_{n+1}^{k+1} = \mathbf{X}_{n+1}^k + \Delta \mathbf{X}_{n+1}^k$$

- $\mathbf{F}^T = [\mathbf{F}_u^T, \mathbf{F}_g^T, \mathbf{F}_w^T, \mathbf{F}_T^T, \mathbf{F}_c^T]$
- $\mathbf{X}^T = [\bar{\mathbf{u}}^T, \bar{\mathbf{p}}_g^T, \bar{\mathbf{p}}_c^T, \bar{\mathbf{T}}^T, \bar{\mathbf{c}}^p{}^T]$
- the time step index : n
- the iteration index : k

## the Jacobian matrix

$$\left. \frac{\partial \mathbf{F}_i}{\partial \mathbf{X}} \right|_{\mathbf{X}^{k+1}} = \left[ \begin{array}{ccccc} \frac{\partial \mathbf{F}_u}{\partial \bar{\mathbf{u}}} & \frac{\partial \mathbf{F}_u}{\partial \bar{\mathbf{p}}_g} & \frac{\partial \mathbf{F}_u}{\partial \bar{\mathbf{p}}_c} & \frac{\partial \mathbf{F}_u}{\partial \bar{\mathbf{T}}} & \frac{\partial \mathbf{F}_u}{\partial \bar{\mathbf{c}}^p} \\ \frac{\partial \mathbf{F}_g}{\partial \bar{\mathbf{u}}} & \frac{\partial \mathbf{F}_g}{\partial \bar{\mathbf{p}}_g} & \frac{\partial \mathbf{F}_g}{\partial \bar{\mathbf{p}}_c} & \frac{\partial \mathbf{F}_g}{\partial \bar{\mathbf{T}}} & \mathbf{0} \\ \frac{\partial \mathbf{F}_w}{\partial \bar{\mathbf{u}}} & \frac{\partial \mathbf{F}_w}{\partial \bar{\mathbf{p}}_g} & \frac{\partial \mathbf{F}_w}{\partial \bar{\mathbf{p}}_c} & \frac{\partial \mathbf{F}_w}{\partial \bar{\mathbf{T}}} & \mathbf{0} \\ \frac{\partial \mathbf{F}_T}{\partial \bar{\mathbf{u}}} & \frac{\partial \mathbf{F}_T}{\partial \bar{\mathbf{p}}_g} & \frac{\partial \mathbf{F}_T}{\partial \bar{\mathbf{p}}_c} & \frac{\partial \mathbf{F}_T}{\partial \bar{\mathbf{T}}} & \mathbf{0} \\ \frac{\partial \mathbf{F}_c}{\partial \bar{\mathbf{u}}} & \frac{\partial \mathbf{F}_c}{\partial \bar{\mathbf{p}}_g} & \frac{\partial \mathbf{F}_c}{\partial \bar{\mathbf{p}}_c} & \frac{\partial \mathbf{F}_c}{\partial \bar{\mathbf{T}}} & \frac{\partial \mathbf{F}_c}{\partial \bar{\mathbf{c}}^p} \end{array} \right]_{\mathbf{X}=\mathbf{X}_{n+1}^k} = \left[ \begin{array}{ccccc} \mathbf{K}_{uu} & \mathbf{K}_{ug} & \mathbf{K}_{uc} & \mathbf{K}_{uT} & \mathbf{K}_{up} \\ \mathbf{K}_{gu} & \mathbf{K}_{gg} & \mathbf{K}_{gc} & \mathbf{K}_{gT} & \mathbf{0} \\ \mathbf{K}_{cu} & \mathbf{K}_{cg} & \mathbf{K}_{cc} & \mathbf{K}_{cT} & \mathbf{0} \\ \mathbf{K}_{Tu} & \mathbf{K}_{Tg} & \mathbf{K}_{Tc} & \mathbf{K}_{TT} & \mathbf{0} \\ \mathbf{K}_{pu} & \mathbf{K}_{pg} & \mathbf{K}_{pc} & \mathbf{K}_{pT} & \mathbf{K}_{pp} \end{array} \right]_{\mathbf{X}=\mathbf{X}_{n+1}^k}$$

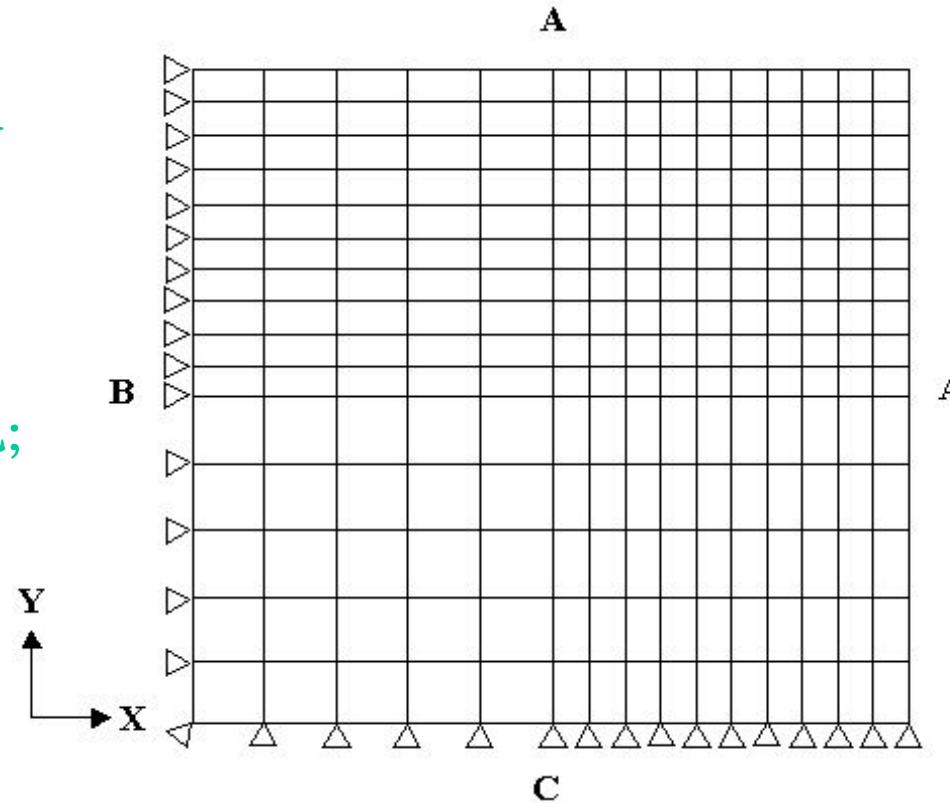


化学-热-湿-气-力学耦合过程一致性切线“刚度”矩阵

# 火灾下混凝土柱典型横截面中的耦合响应模拟

## A - 混凝土柱与大气的边界与热荷载作用面

- 混凝土固体骨架变形;
- 含基质成份的不混溶水流;
- 不混溶混合气流;
- 热流;



$$T = 298 + 0.5t$$

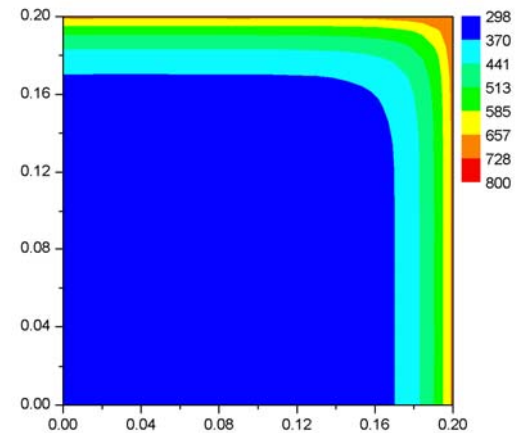
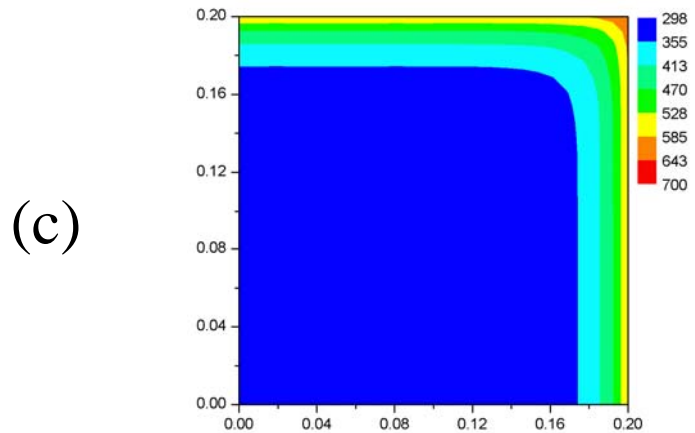
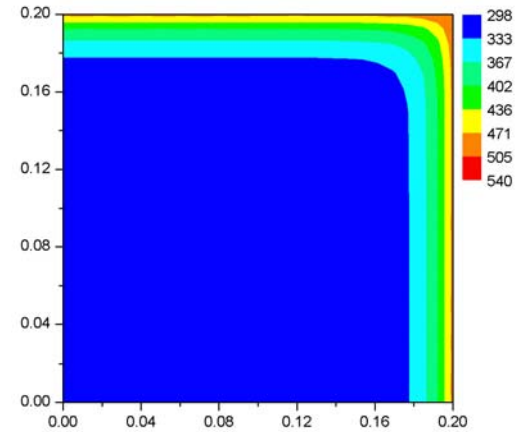
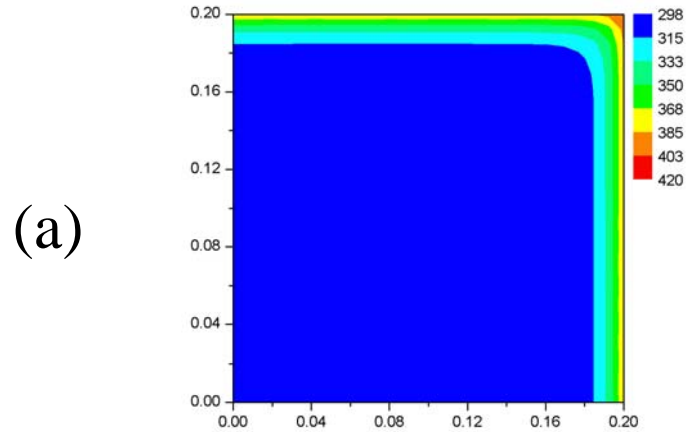
$$T_{\max} = 800\text{K}$$

$$S_l^0 = 0.47$$

混凝土横截面的1/4

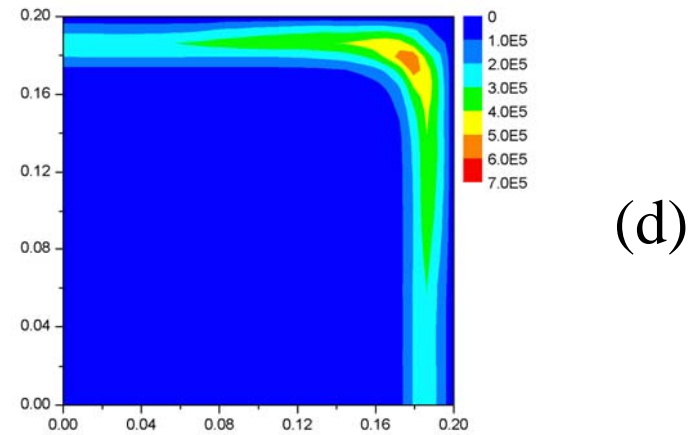
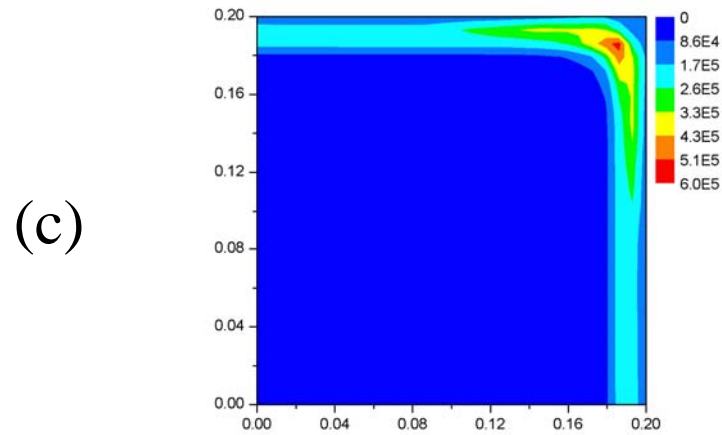
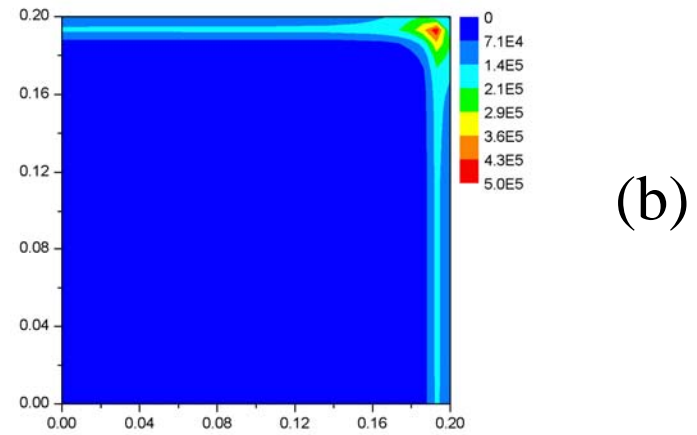
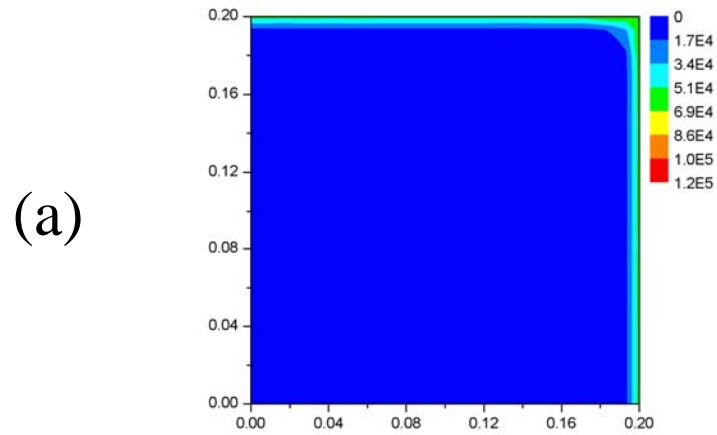
不同时刻下 (a) 4 min. (b) 8 min. (c) 12 min. (d) 16 min.  
混凝土柱中物理量数值模拟结果





Temperature distributions in concrete column at time levels:

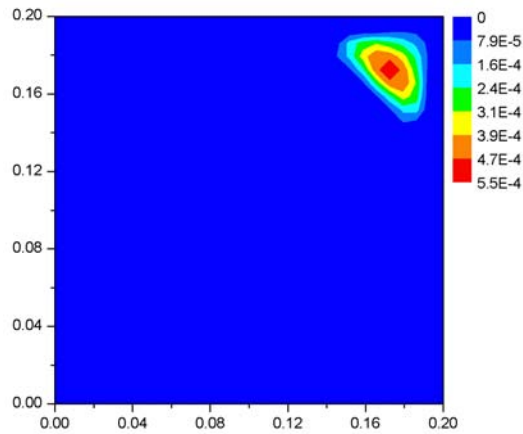
(a) 4 min. (b) 8 min. (c) 12 min. (d) 16 min.



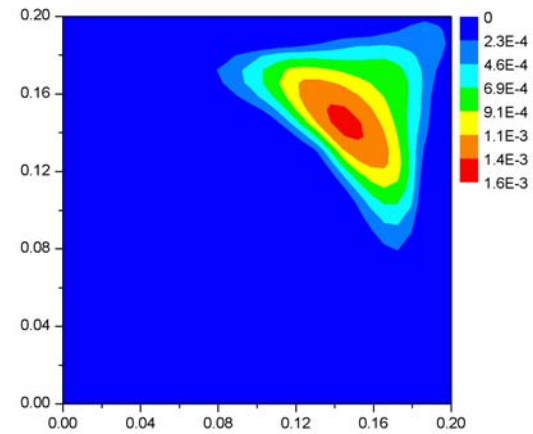
Vapor pressure distributions (Pa) in concrete column at time levels:

(a) 4 min. (b) 8 min. (c) 12 min. (d) 16 min.

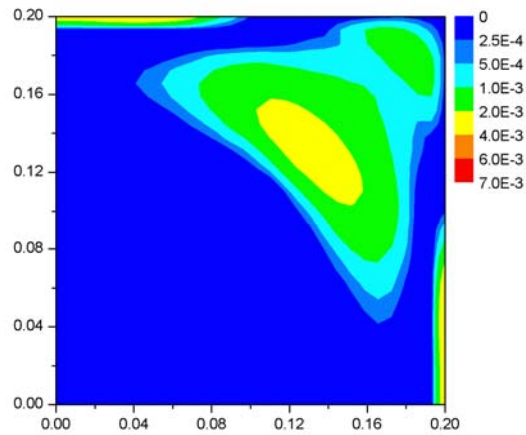
(a)



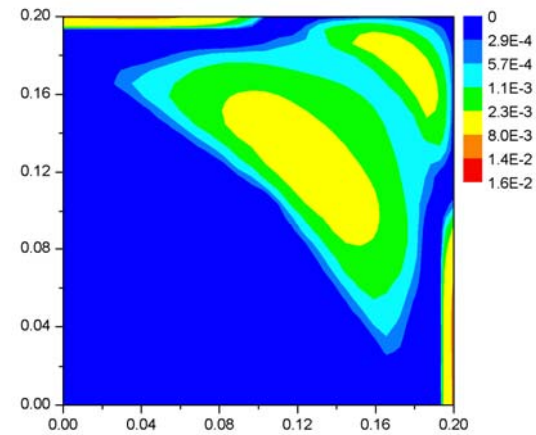
(b)



(c)

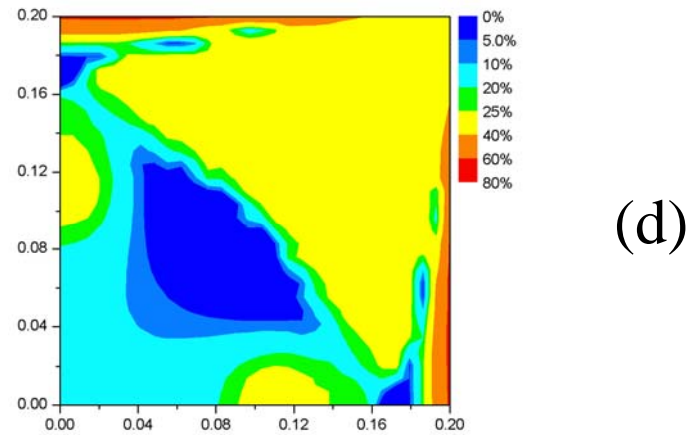
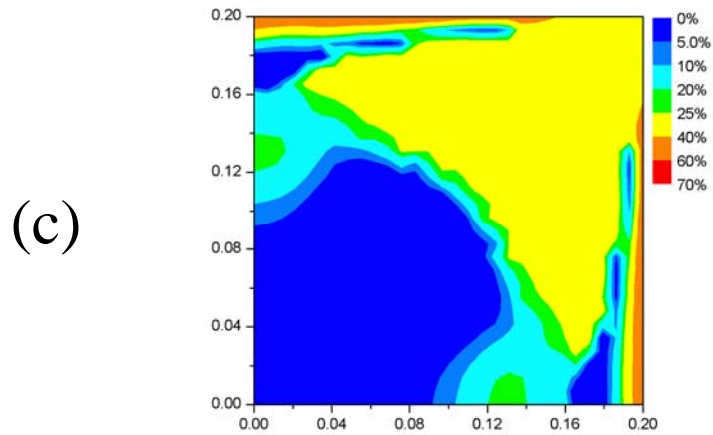
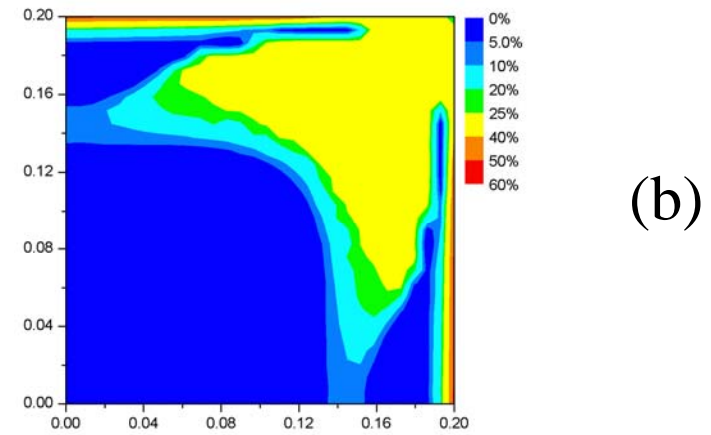
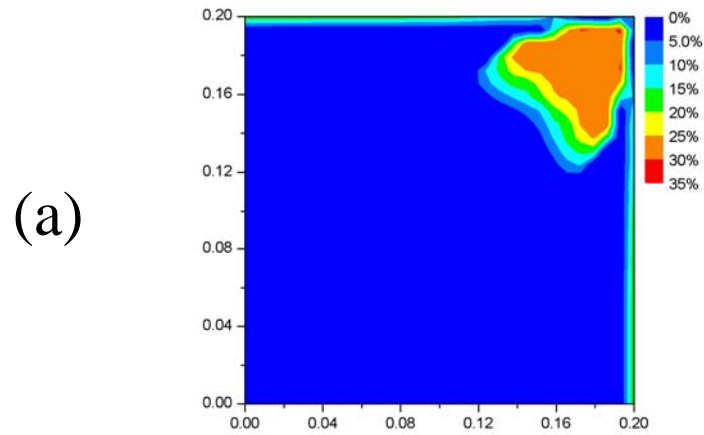


(d)



Equivalent plastic strain distributions in concrete column at time levels:

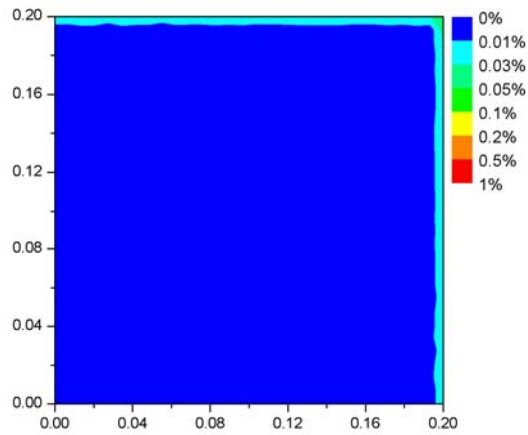
(a) 4 min. (b) 8 min. (c) 12 min. (d) 16 min.



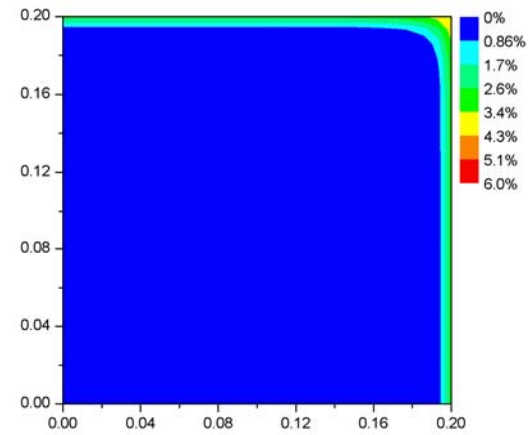
Mechanical damage distributions in concrete column at time levels:

(a) 4 min. (b) 8 min. (c) 12 min. (d) 16 min.

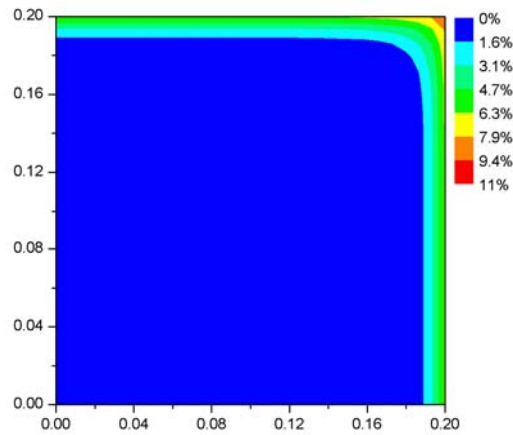
(a)



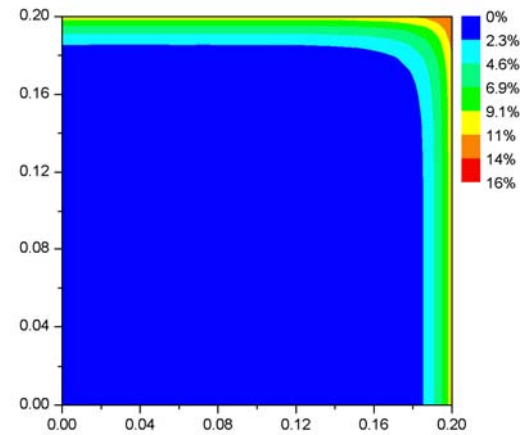
(b)



(c)

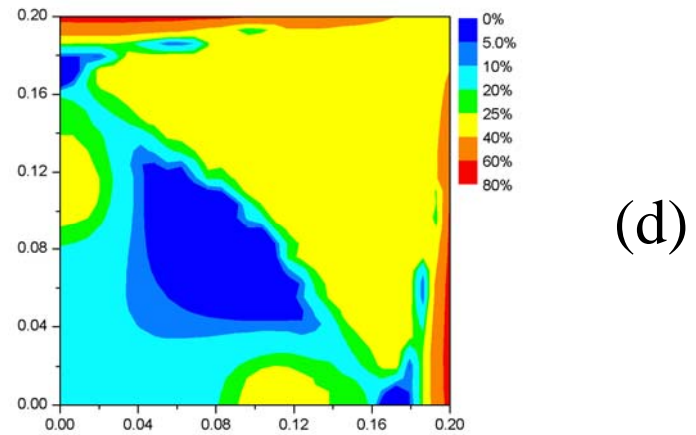
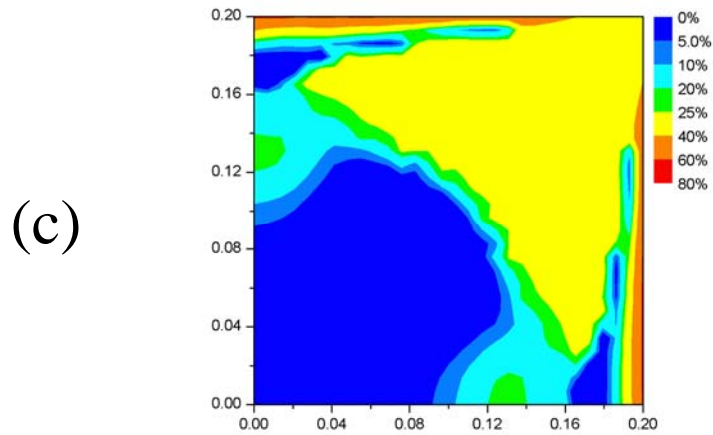
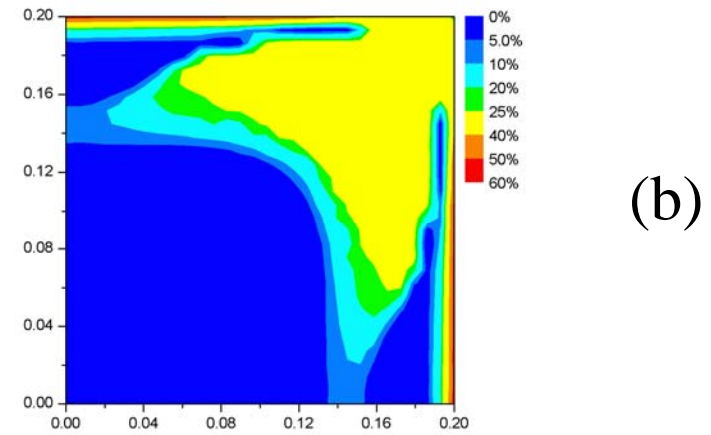
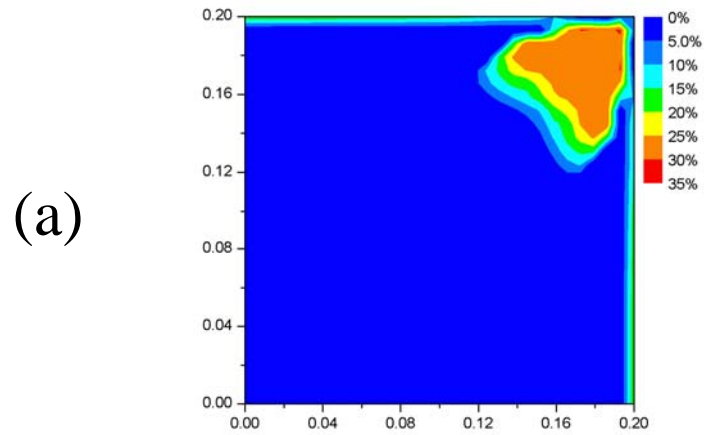


(d)



Chemical damage distributions in concrete column at time levels:

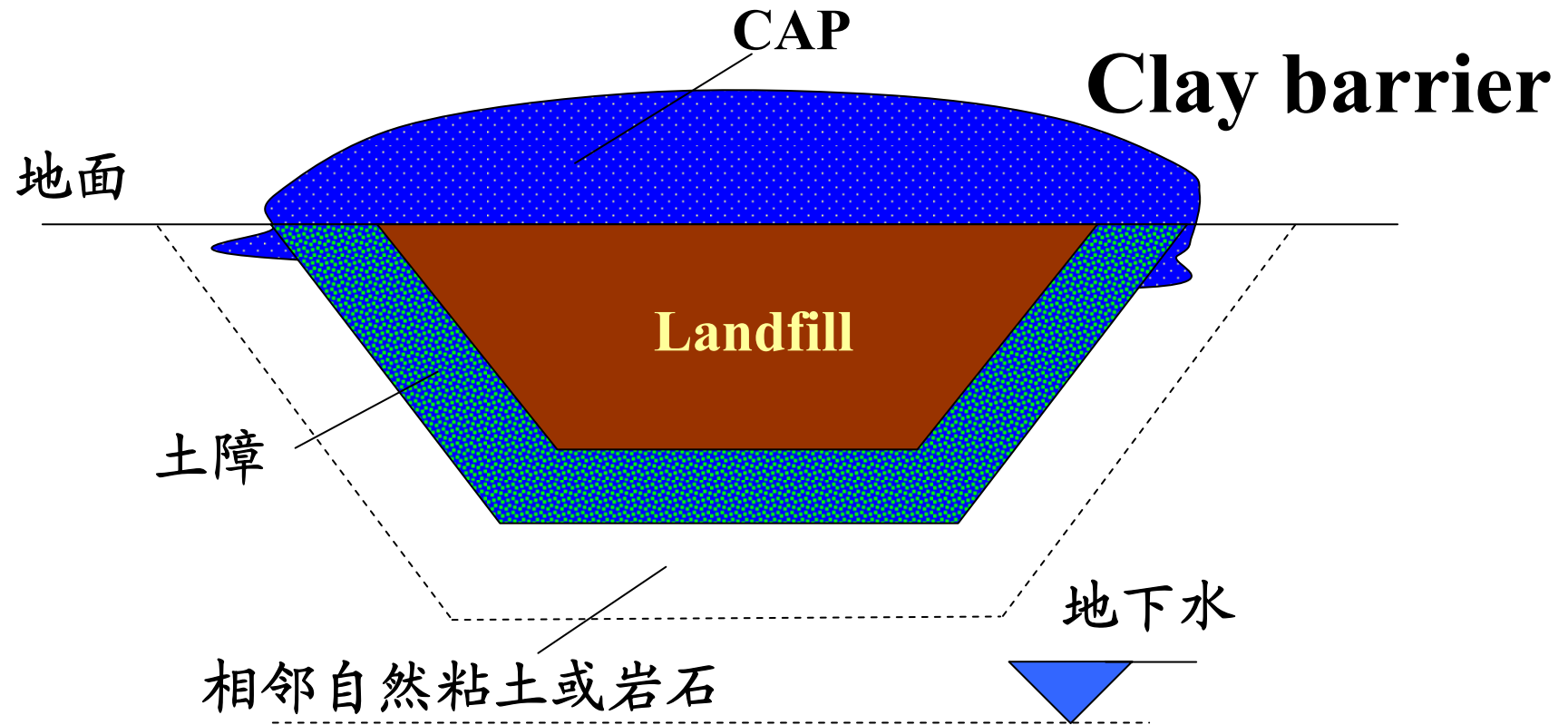
(a) 4 min. (b) 8 min. (c) 12 min. (d) 16 min.



Total damage distributions in concrete column at time levels:

(a) 4 min. (b) 8 min. (c) 12 min. (d) 16 min.

# 非饱和土中热-湿-气-力学-传质 (CTHM) 全耦合数学模型与数值方法

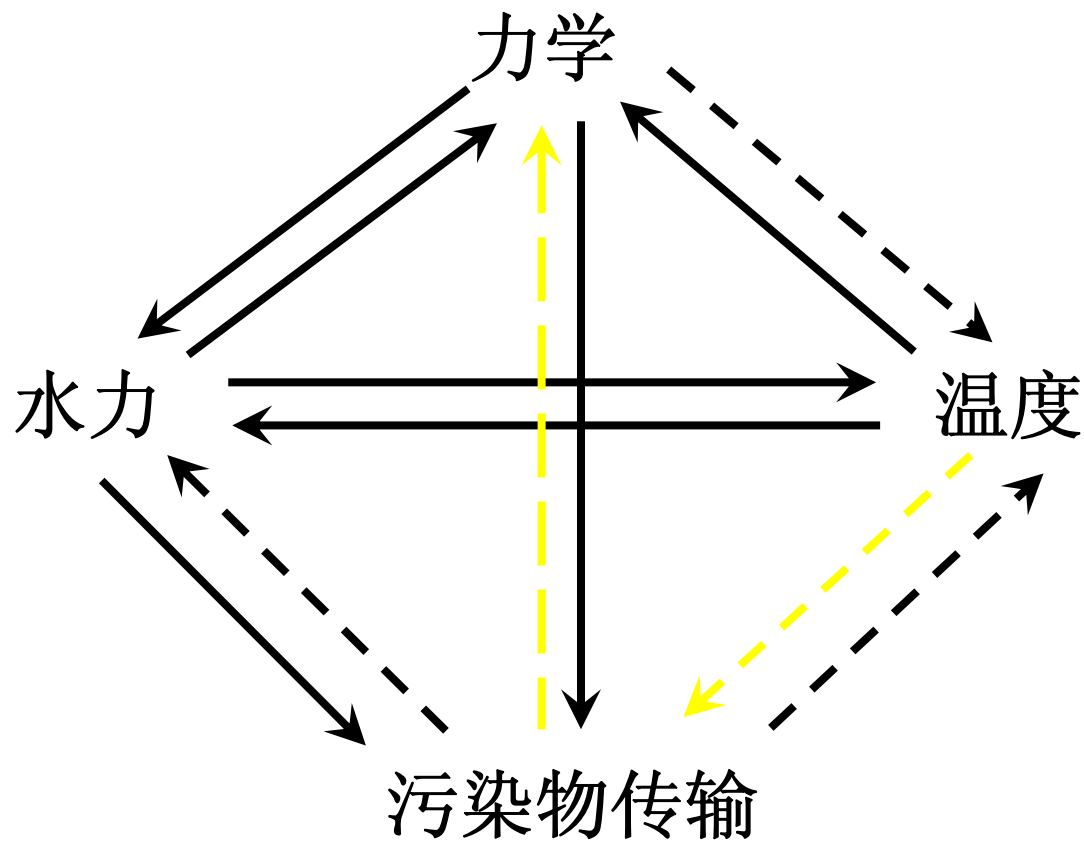


## ➤ 背景问题:

工程土障中热-水力-力学-(污染物)传质耦合过程数值模拟

# 科学问题

## 耦合效应

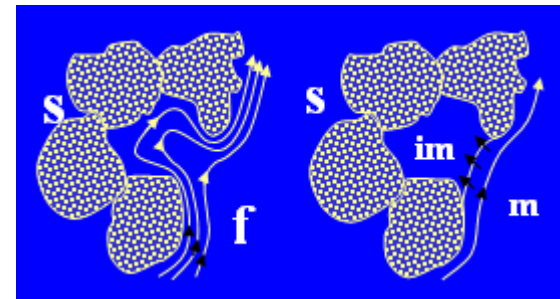
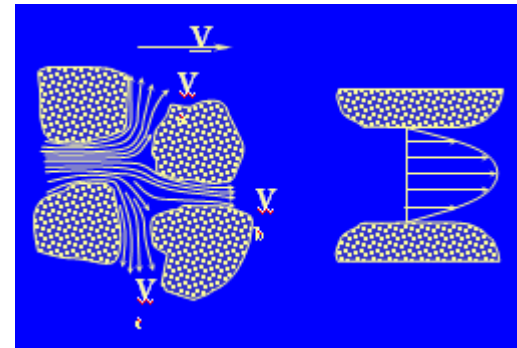




# 分模型- 污染物随地下水传输模型 (宏观、唯象)

## 控制机制

- 对流
- 分子扩散
- 机械逸散
- 吸附: 平衡吸附 (可逆)  
非平衡吸附 (不可逆)
- 蜕变
- 不动水效应



- Li Xikui, Cescotto S, Thomas HR. Finite element method for contaminant transport in unsaturated soils. [ASCE Journal of hydrologic Eng., 1999, 4: 265-274.](#)
- Li Xikui, Wu Wenhua, Cescotto S. Contaminant transport with non-equilibrium processes in unsaturated soils and implicit characteristic Galerkin scheme. [Int. J. for Numerical and Analytical Methods in Geomechanics, 2000,24: 219-243.](#)
- Wu Wenhua, Li Xikui, Charlier R, Collin F. A thermo-hydro-mechanical constitutive model and its numerical modelling for unsaturated soils [Computers & Geotechnics, 2004, 31: 155-167.](#)

# 对流扩散方程的数值方法

## 对流扩散方程的一般形式

$$L_c = \frac{\partial \phi}{\partial t} + \mathbf{u}^T \nabla \phi + A\phi - B \operatorname{div}(\mathbf{D} \nabla \phi) + Q = 0$$

## 对流扩散方程的非自伴随特征



标准 Galerkin 有限元方法的 (空间) 离散化过程不再有效

## 数值求解方案?

- 发展了基于特征 Galerkin 法和精细积分的隐式无条件稳定算法, 允许实现对 大 Courant 数 (大时间步长) 和大 Peclet 数 (对流占优) 的对流扩散问题数值模拟

Li Xikui, Wu Wenhua, O.C.Zienkiewicz. **Implicit characteristic Galerkin method for convection - diffusion equations.**

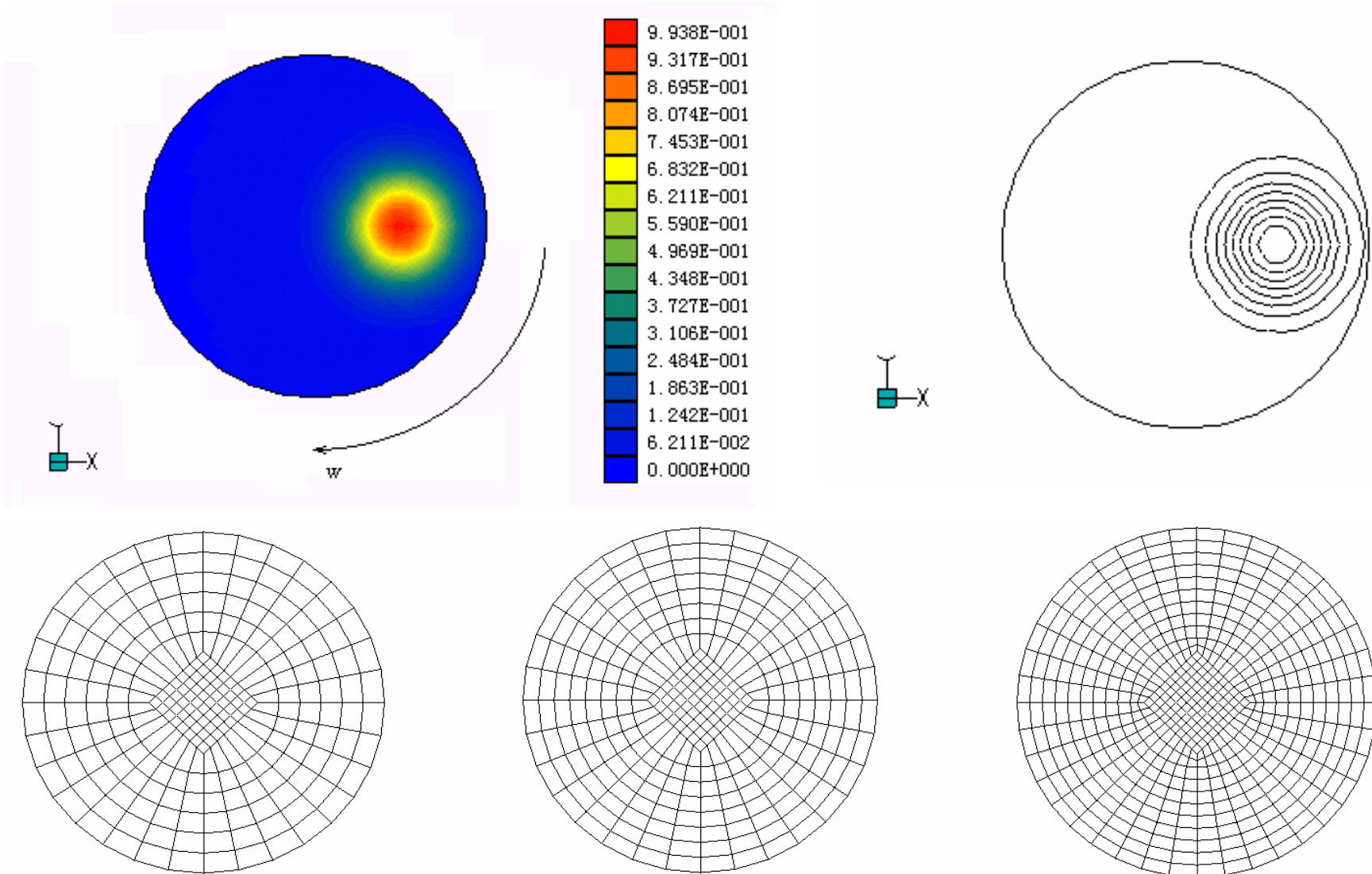
**Int. J. for Numerical Methods in Eng, 2000, 47: 1689-1708.**

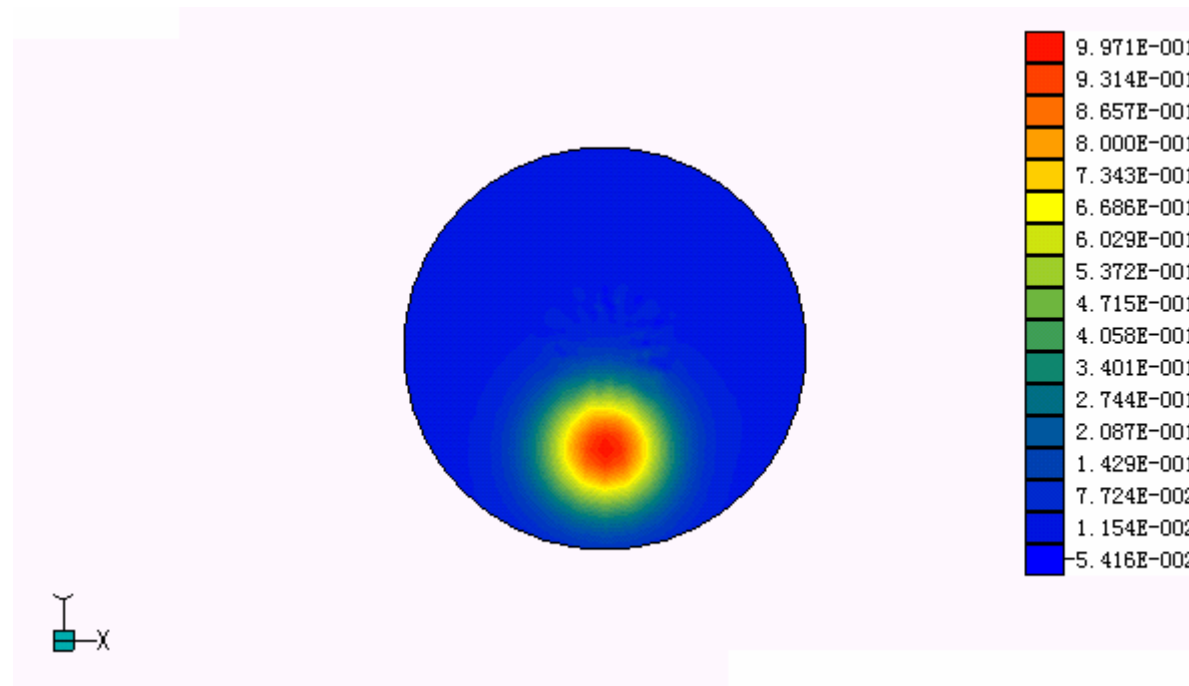
# ”The rotating hill“问题

初值: a hill located and described as

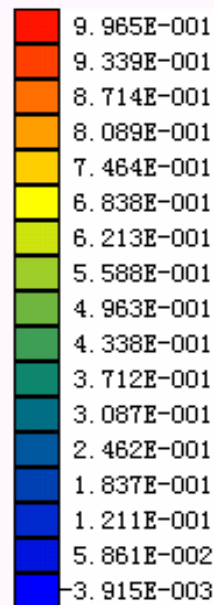
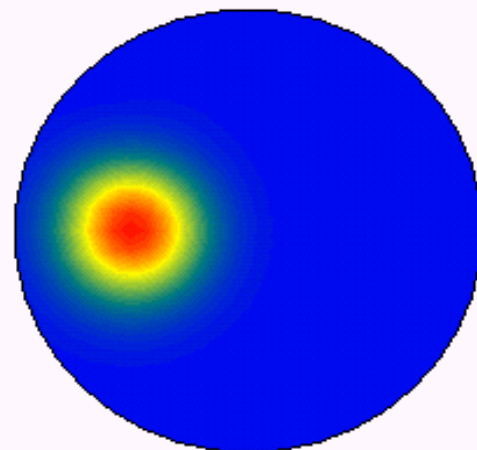
$$\mathbf{x}_0 = [0.5, 0.0]^T \quad \phi(\mathbf{x})|_{t=0} = \exp(-10(\mathbf{x} - \mathbf{x}_0)^2)$$

$$\phi(\mathbf{x}_0)|_{t=0} = 1.0$$

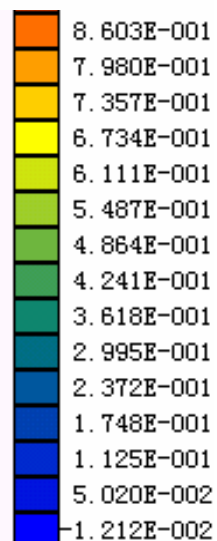
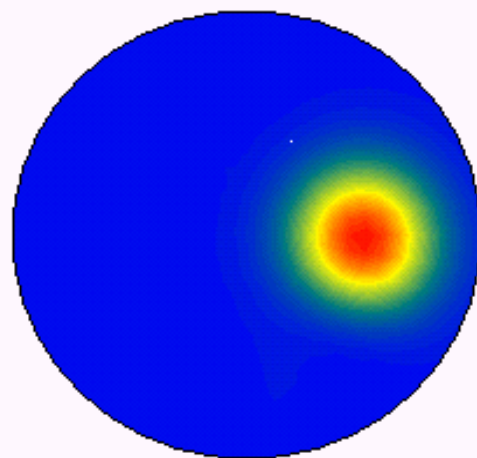




**Moving contour plots of the exponential hill**  
(25步, 时间步长:  $\Delta t = \pi/10$  )



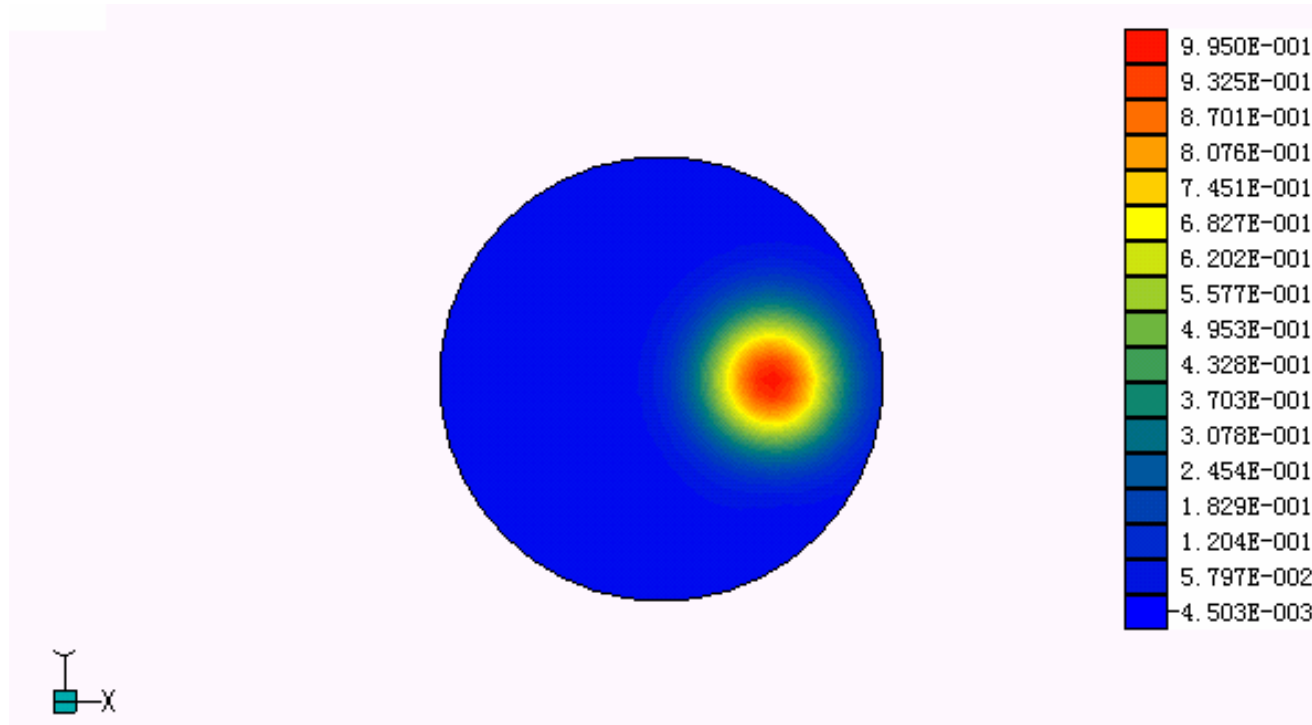
(第3步)



Contour plots of the exponential hill

(时间步长  $\Delta t = \pi$  )

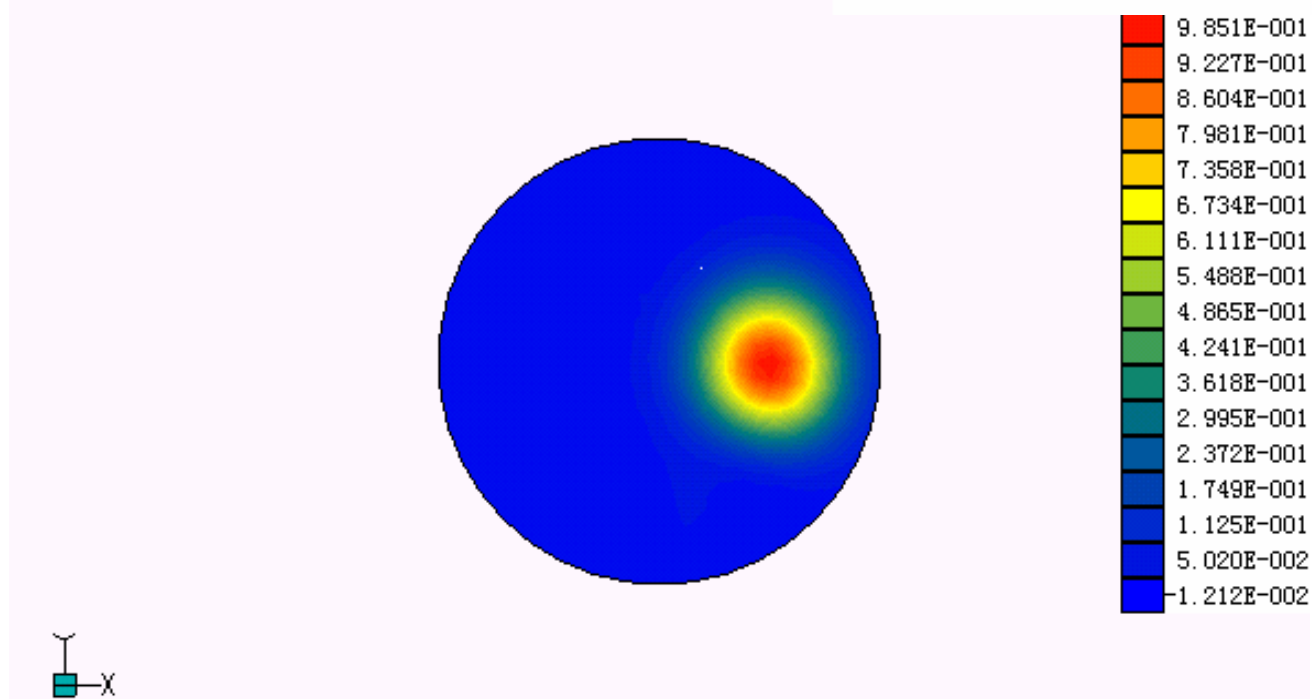
(第10步)



(第2步)

**Contour plots of the exponential hill**

(时间步长  $\Delta t = 2\pi$ )



(第5步)

# 离散颗粒模型及离散元方法

## 模型特点:

- ▶ 基于不同半径接触圆形颗粒间运动学分析的颗粒间位移-力“弹塑性”本构关系，对滚动力、滚动矩给予了特别关注；
- ▶ 以圆形颗粒组成的“颗粒簇”模拟非圆形颗粒；
- ▶ 颗粒材料非线性与破坏在离散颗粒模型中的体现：
  - 颗粒间位移-力“弹塑性”本构关系
  - 颗粒接触拓扑（接触力链结构）随时间的变化
  - 以颗粒簇的解簇模拟颗粒破碎
- ▶ 对饱和颗粒材料的离散-连续模型  
固体颗粒集合体以离散颗粒模型描述并以离散单元方案求解，间隙水分别基于不可压缩N-S方程与Darcy定律两种平行的连续介质模型描述，并以特征线SPH方案描述和求解流体质点相对固体颗粒的运动

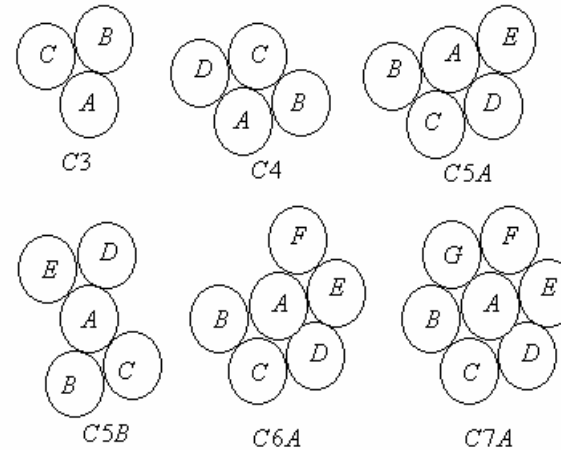
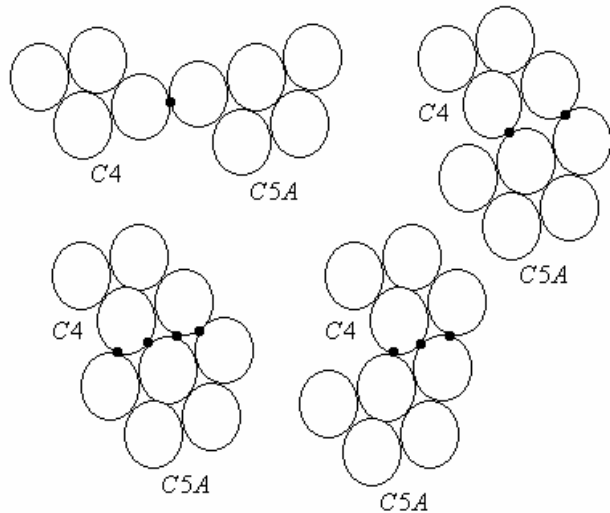
# 颗粒分级模型

一级基本颗粒及一级颗粒簇

二级基本颗粒及二级颗粒簇

不可破碎的物质颗粒

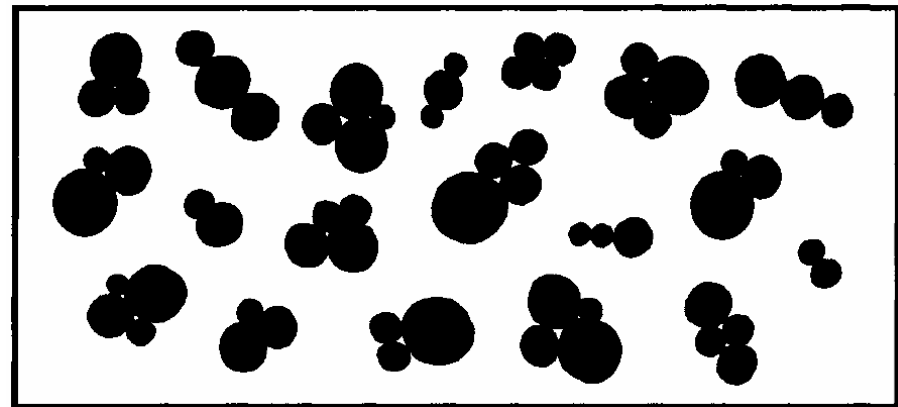
至少有两个接触点



若干典型颗粒簇

颗粒簇具有模拟多个接触点的能力

以颗粒簇模拟非圆形颗粒





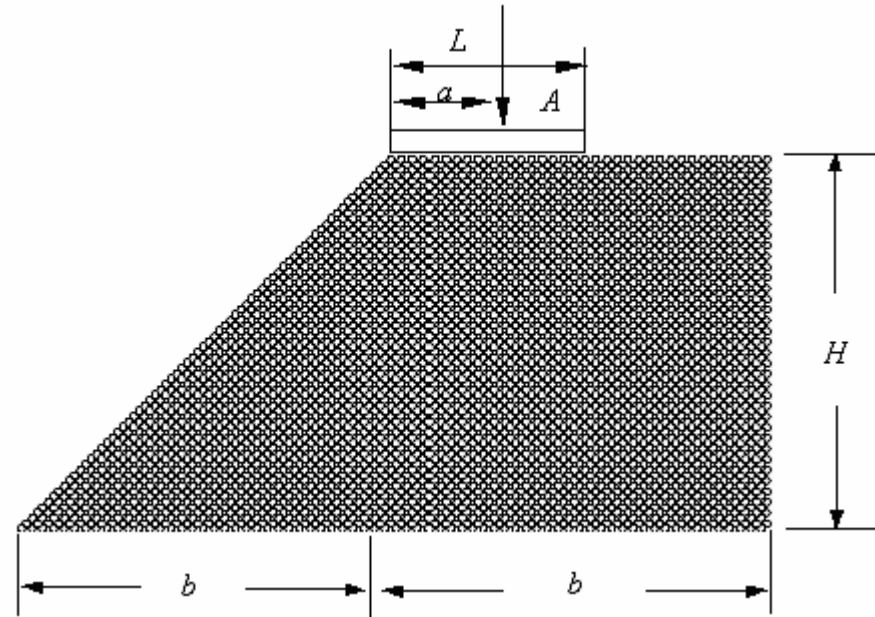
# 破碎模式

颗粒类型	破坏模式					
	模式1	模式2	模式3	模式4	模式5	模式6
独立颗粒 C7A	$1P + C6A$	$2P + C5A$	$2P + C5B$	$3P + C4$	$4P + C3$	$7P$
颗粒簇 C6A	$1P + C5A$	$1P + C5B$	$2P + C4$	$3P + C3$	$6P$	
颗粒簇 C5A	$1P + C4$	$2P + C3$	$5P$			
颗粒簇 C5B	$2P + C3$	$5P$				
颗粒簇 C4	$1P + C3$	$4P$				
颗粒簇 C3	$3P$					

## 边坡破坏算例

显示离散颗粒模型  
捕捉颗粒材料边坡  
不同破坏模式的能力

- 滑坡
- 泥石流
- 压裂



$b=50\text{cm}$ ,  $H=50\text{cm}$ ,  $L=25\text{cm}$ ,  $a=15\text{cm}$

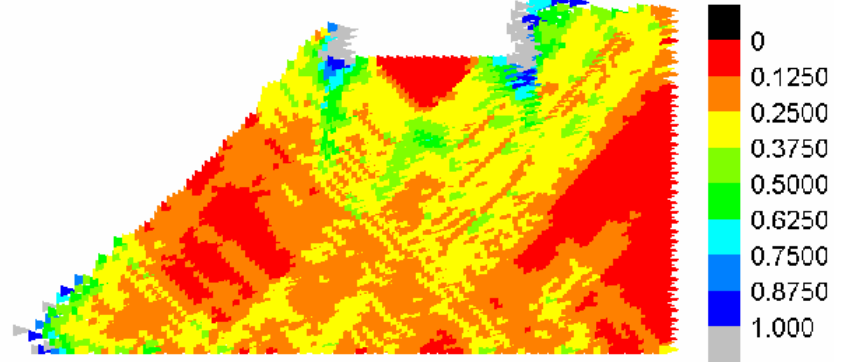
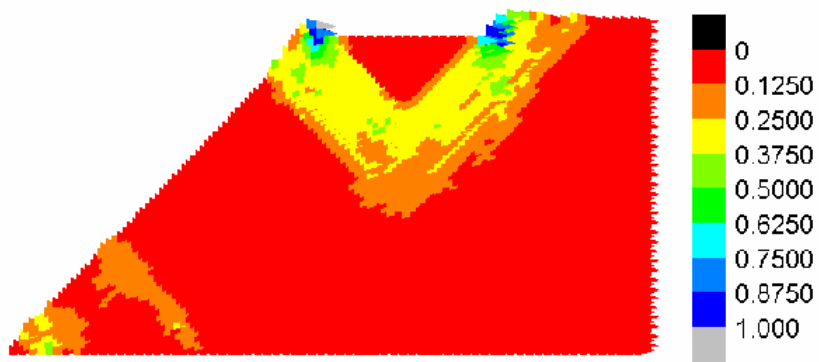
3710个, 半径5mm, 规则方式排列

Xikui Li, Xihua Chu, Y.T.Feng

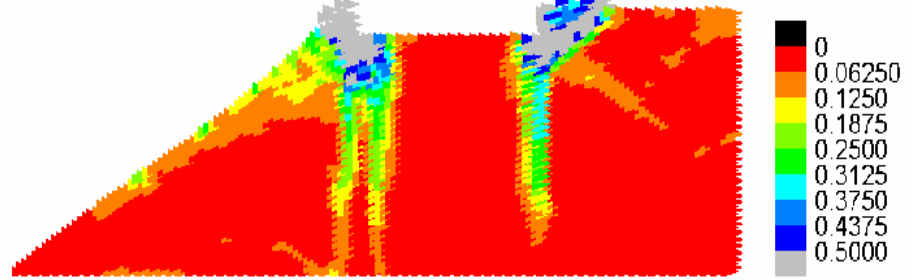
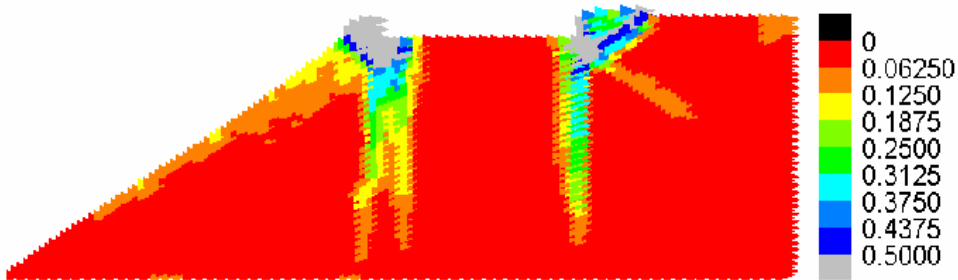
A discrete particle model and numerical modelling  
of the failure modes of granular materials,

[Engineering Computations, 22\(8\): 894-920,2005.](#)

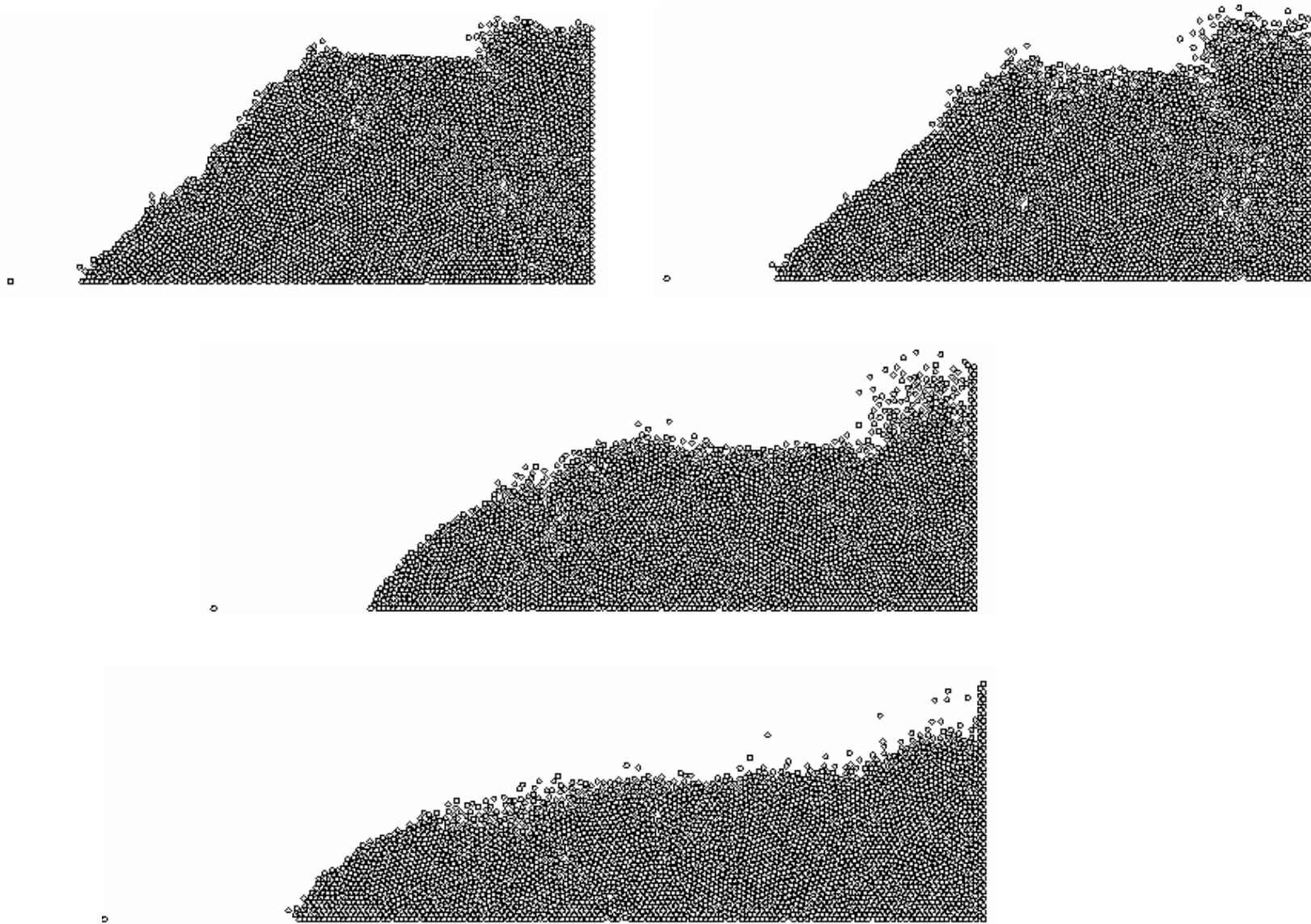
# 滑坡破坏模式



# 压裂破坏模式



# 泥石流破坏模式



# 饱和颗粒材料液固耦合模拟

## 含液颗粒材料固相颗粒模型的运动方程

二维情况典型固体颗粒  $a$  的运动方程

质量  $\rightarrow$  
$$m_a \frac{d\mathbf{v}_a}{dt} = \sum_{j=1}^{k_a} \mathbf{f}^{cj} + \mathbf{f}^{fs} + m_a \mathbf{g} + \mathbf{f}^e$$

转动惯量  $\rightarrow$  
$$I_a \frac{d\omega_a}{dt} = \sum_{j=1}^{k_a} (\mathbf{r}_a^j \times \mathbf{f}^{cj} + M_r^j)$$

流固耦合作用

◆ 所建议模型可作为饱和颗粒材料两尺度模型分析的细尺度部分

● Xikui Li, Xihua Chu, D.C. Sheng

A saturated discrete particle model and characteristic based SPH method in granular materials.

Int. J. Numerical Methods in Eng, 72: 858-882, 2007.

# 饱和颗粒材料液固耦合模拟-间隙水连续介质描述

## 平均N-S方程描述

间隙水模拟为无粘流体，以平均变量描述的量方程与连续方程可写为

$$\frac{\partial(n\rho_f)}{\partial t} + \frac{\partial(n\rho_f u_i)}{\partial x_i} = 0$$

$$\frac{\partial(n\rho_f u_i)}{\partial t} + \frac{\partial(n\rho_f u_i u_j)}{\partial x_j} = -n \frac{\partial p}{\partial x_i} + n F_i^{sf}$$

不可压缩时  $\frac{du_i}{dt} = -\frac{1}{\rho_f} \left( \frac{\partial p}{\partial x_i} - F_i^{sf} \right)$

固体对流体的作用： $F_i^{sf} = F_i^{sf}(v_i - u_i, d, n, \mu, \rho \dots)$

$$\frac{du_i}{dt} = \frac{\partial u_i}{\partial t} + \frac{\partial u_i}{\partial x_j} u_j$$

颗粒体积

根据牛顿第三定律，流体对固体颗粒的作用为

$$\mathbf{f}^{fs} = -n \mathbf{F}^{sf} V^p / (1-n)$$

## Darcy定律描述

# 饱和颗粒材料液固耦合模拟-间隙水连续介质描述

## 平均N-S方程描述与Darcy描述的关系

1. 间隙水为不可压缩流体时  $\frac{du_i}{dt} = -\frac{1}{\rho_f} \left( \frac{\partial p}{\partial x_i} - F_i^{sf} \right)$

2. 忽略间隙水的动力影响

$$\frac{du_i}{dt} = -\frac{1}{\rho_f} \left( \frac{\partial p}{\partial x_i} - F_i^{sf} \right) = 0 \quad \longrightarrow \quad \frac{\partial p}{\partial x_i} = F_i^{sf}$$

3. 等价的流固耦合作用，即  $F_i^{sf} = \bar{F}_i^{sf} = -k_{ij}^{-1} w_j$

在满足以上三个条件前提下，平均N-S方程退化为Darcy定律

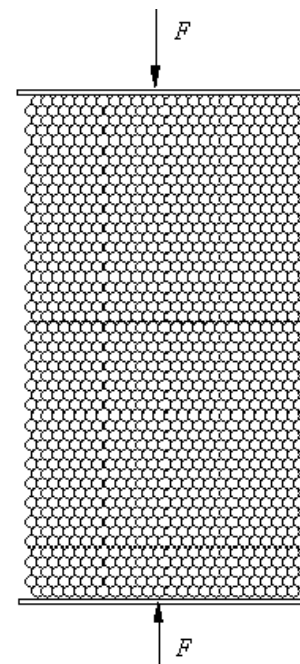
$$w_i = -k_{ij} \frac{\partial p}{\partial x_j}$$

# 数值算例

## 问题描述

1249个半径为5mm的  
固体颗粒规则排列

Fig4.1



## 固相参数

$$\rho_s = 2000 \text{ kg m}^{-3}$$

$$k_n = 6.0 \times 10^8 \text{ Nm}^{-1}$$

$$k_s = 4.0 \times 10^8 \text{ Nm}^{-1}$$

$$\mu_s = 0.5$$

$$k_r = 2.0 \times 10^3 \text{ Nm}^{-1}$$

$$k_\theta = 700 \text{ N m rad}^{-1}$$

$$\mu_r = 0.5$$

$$\mu_\theta r = 1.0 \times 10^{-4} \text{ m}$$

$$c_n = c_s = c_r = c_g = 0$$

## 液相参数

$$\rho_w = 1.0 \times 10^3 \text{ kg m}^{-3}$$

$$\mu = 1.01 \times 10^{-3} \text{ Nm}^{-2} \text{ s}$$

$$h = 12.5 \text{ mm}$$

$$25 \times 44.3 \text{ cm}^2$$

## 边界条件

刚性板的加载速度为  $6.18 \times 10^{-2} \text{ m/s}$



# 数值算例

## Case I

只有刚性板加载

初始孔隙水压力为

$$1.0 \times 10^5 \text{ pa}$$

## Case II

板加载, 同时上边界注水

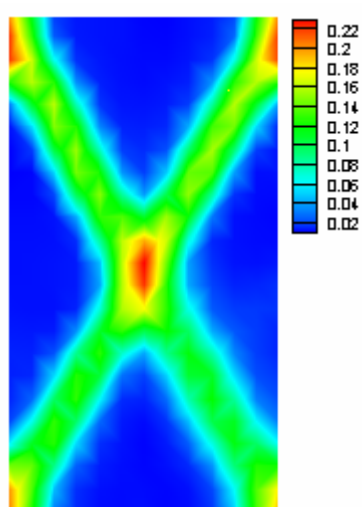
$$w_y = 1.3 \times 10^{-1} \text{ m/s}$$

初始孔隙水压力为

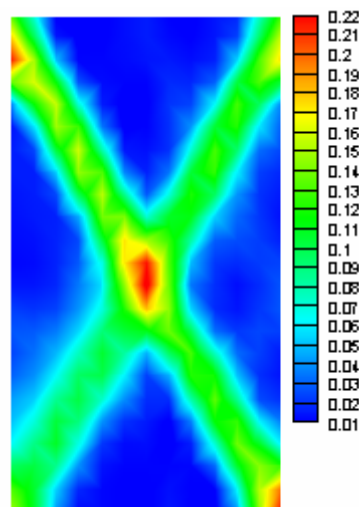
$$1.0 \times 10^5 \text{ pa}$$

# Case I: 名义有效应变

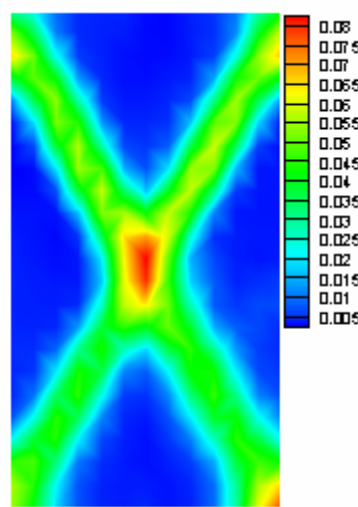
# 名义体积应变



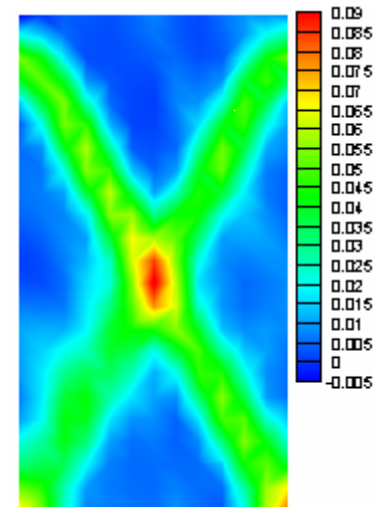
N-S



Darcy



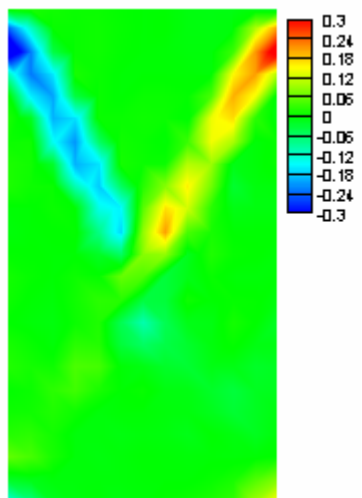
N-S



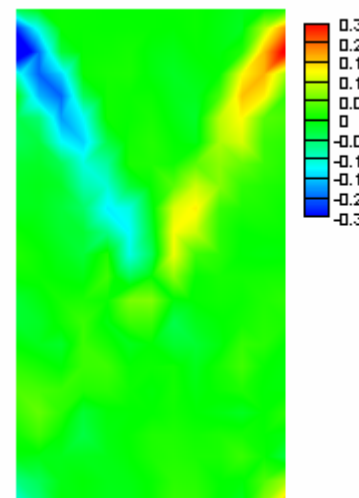
Darcy

## 颗粒旋转

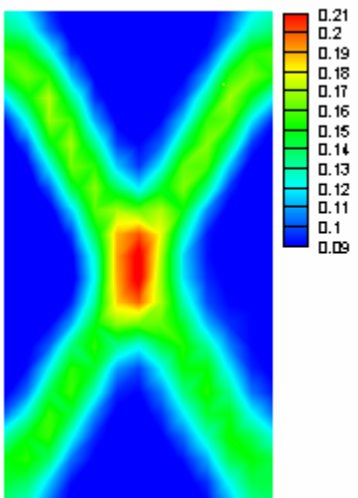
## 孔隙度



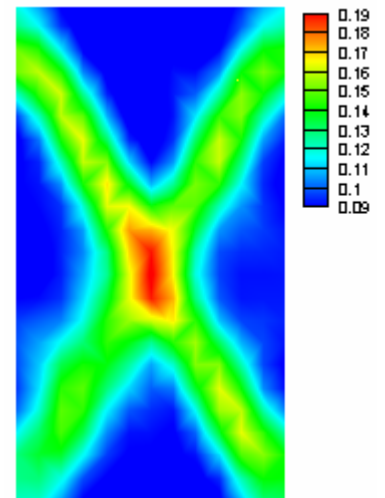
N-S



Darcy



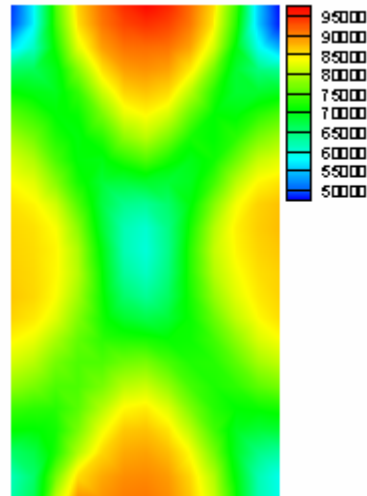
N-S



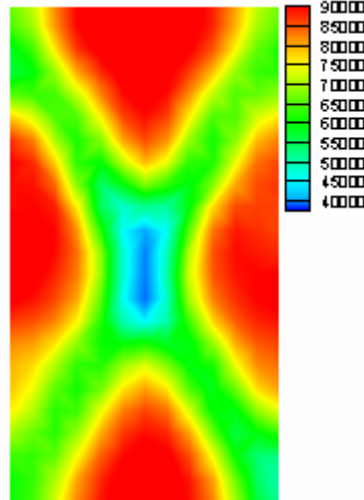
Darcy

# Case I:

## 孔隙水压力

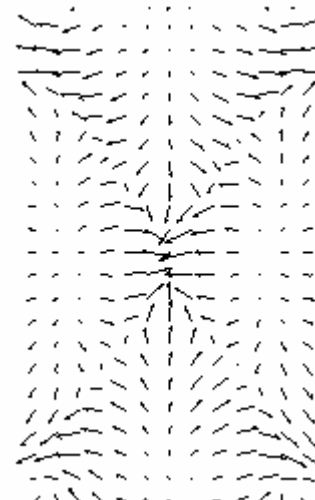


### N-S

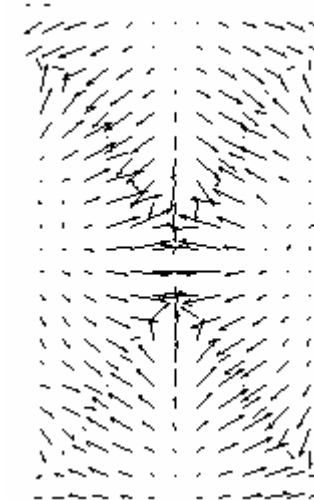


### Darcy

## 孔隙水流线



### N-S

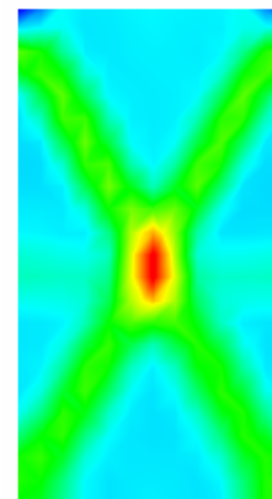
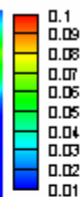
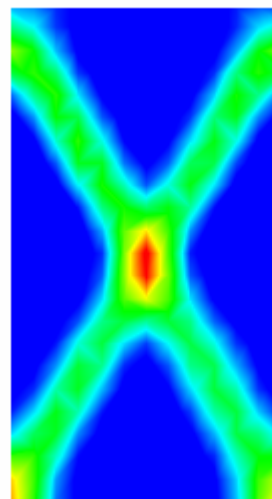
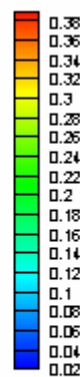
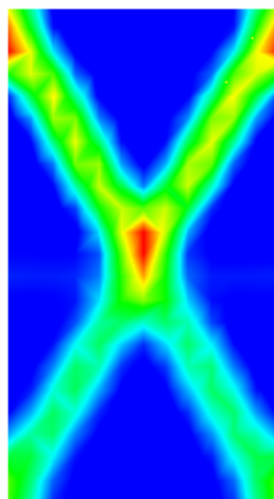
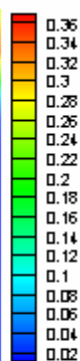
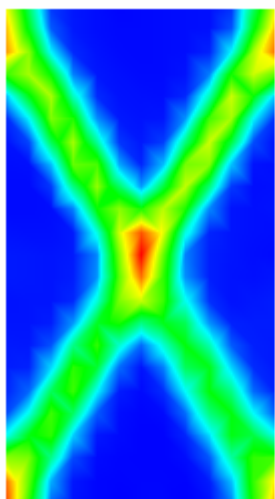


### Darcy

- 有效应变，体积应变，颗粒旋转，孔隙度变化集中在一个狭窄带、（剪切带）内发展，即出现局部化现象
- 孔隙水流线图论证了孔隙流体主要地流向剪切带，即孔隙度显著增大的区域

## Case II: 名义有效应变

## 名义体积应变



N-S

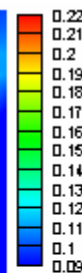
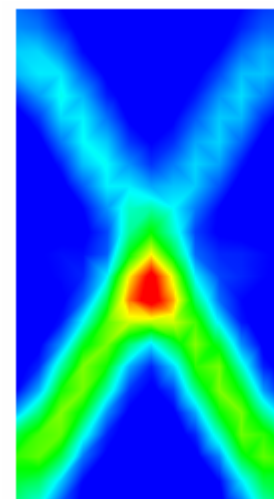
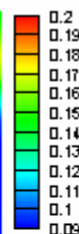
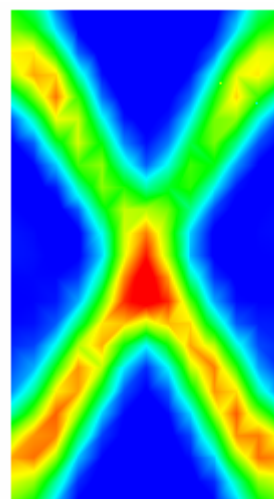
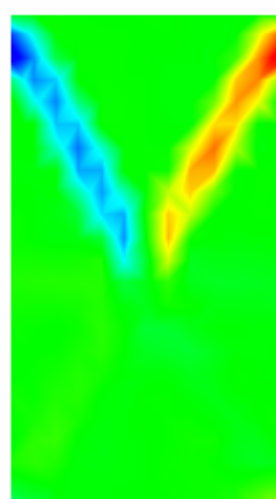
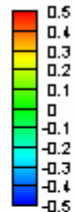
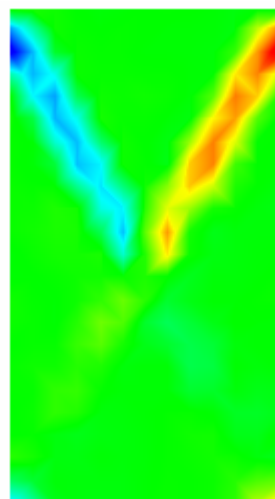
Darcy

N-S

Darcy

颗粒旋转

孔隙度



N-S

Darcy

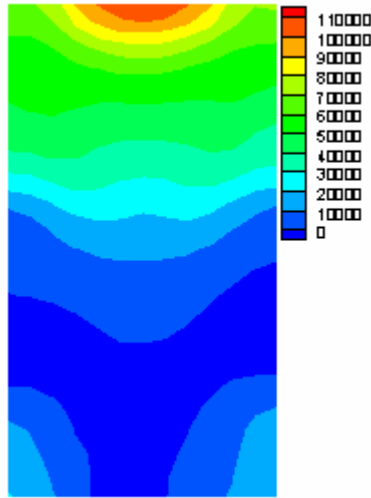
N-S

Darcy

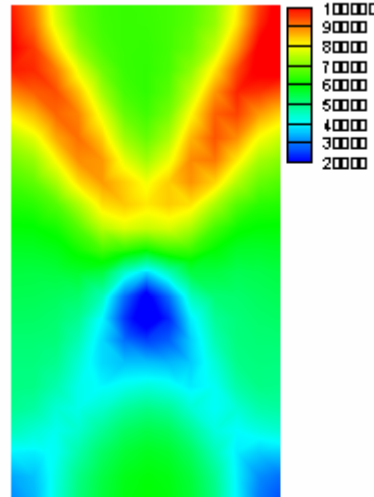
## Case II:

孔隙水压力

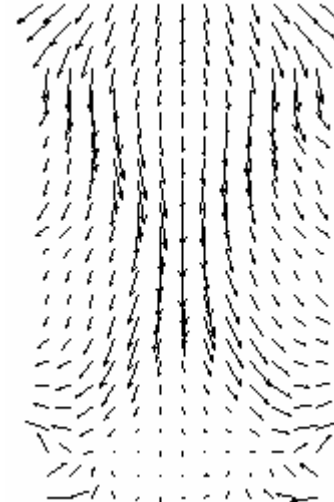
孔隙水流线



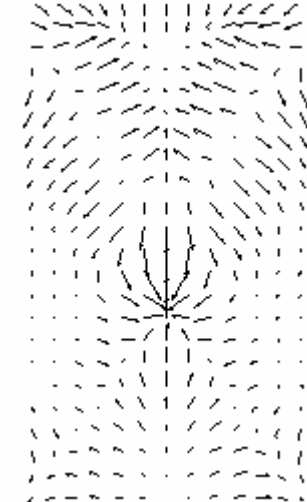
N-S



Darcy



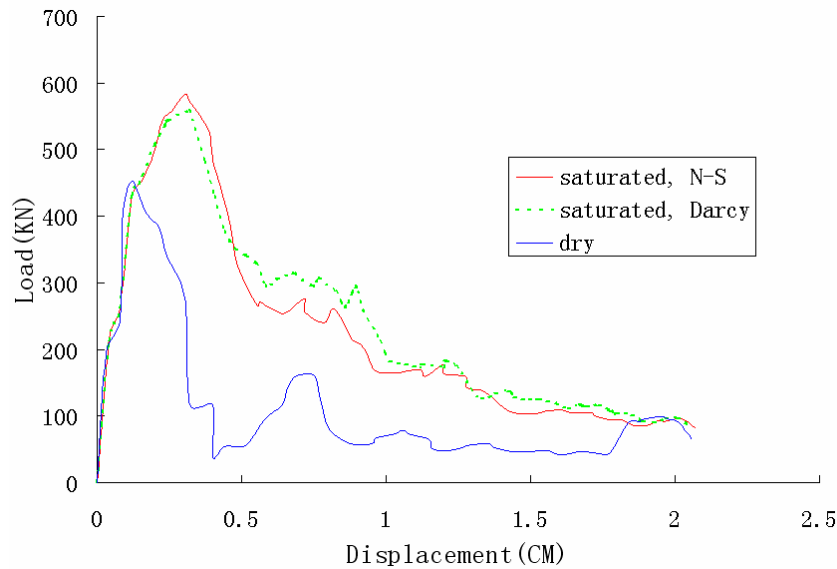
N-S



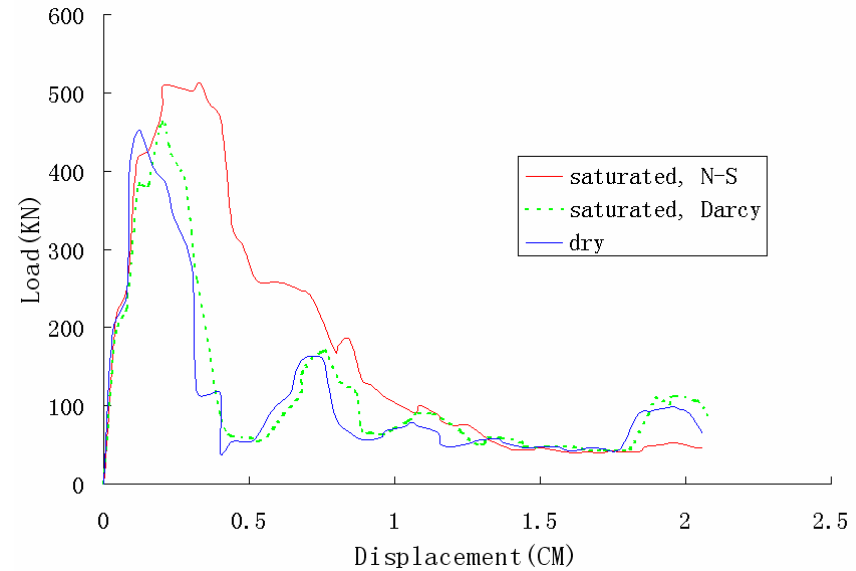
Darcy

- 注水对孔隙水流动有显著影响
- 数值结果重现了实验结论: 即孔隙水流速较高时, Darcy定律有较大误差

# 荷载-位移曲线比较



CASE I (仅机械荷载)



CASE II(机械荷载+降雨)

1. 论证了软化现象，孔隙水承担部分载荷，并延迟了软化行为的出现
2. 由于惯性影响N-S方程给出的承载力估值较高
3. 注水模拟了降雨过程，结果再现了它降低了结构的承载能力
4. 注水加速了破坏过程

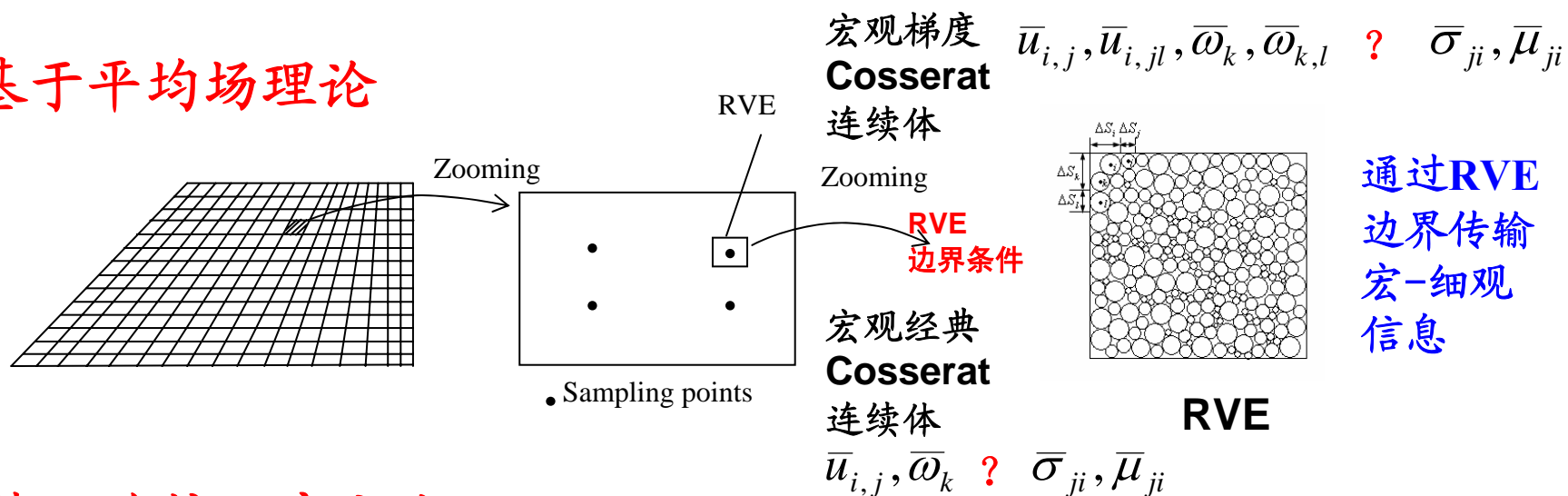
# 颗粒材料多尺度模型

- 颗粒材料多尺度（多分辨率）力学特点  
(Multiresolution Mechanics, W.K.Liu, NU,USA)  
结合材料科学与工程设计
  - 基于细观颗粒行为的宏观连续体响应
  - 理解与颗粒微结构响应相对应的连续体响应
  - 发展颗粒材料多分辨尺度控制方程、材料本构律和相应的一致性(concurrent)计算方法
  - 颗粒细观尺度的材料破坏
- 多尺度力学是为发现具有创新性的新科学与工程的关键学科  
需要发展新的离散颗粒集合体-Cosserat连续体力学的理论框架、数值算法、计算体系。

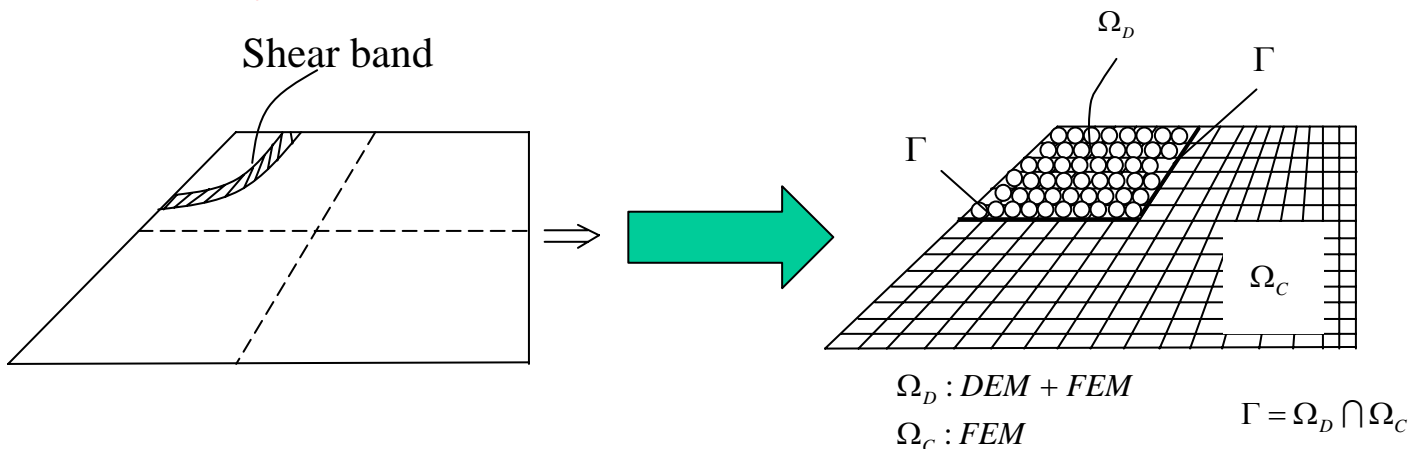
# 颗粒材料多尺度模型

## 宏、细尺度计算均匀化的两个基本途径

### 基于平均场理论



### 基于连接尺度方法





# 离散颗粒集合体 - 梯度增强Cosserat连续体模型的细-宏观均匀化方法

## 提出非均质Cosserat连续体的Hill定理

(Xikui Li et al. *Acta Mechanica Sinica*, 2009; *I J Solids Structures*, 2010)

细观应变能

宏观应变能

$$\overline{\sigma_{ji} \varepsilon_{ji}} + \overline{\mu_{ji} \kappa_{ji}} - \overline{\sigma_{ji}} \overline{\varepsilon_{ji}} - \overline{\mu_{ji}} \overline{\kappa_{ji}}$$

$$= \frac{1}{V} \int_S (n_k \sigma_{ki} - n_k \overline{\sigma}_{ki}) (u_i - \overline{u}_{i,j} x_j) dS +$$

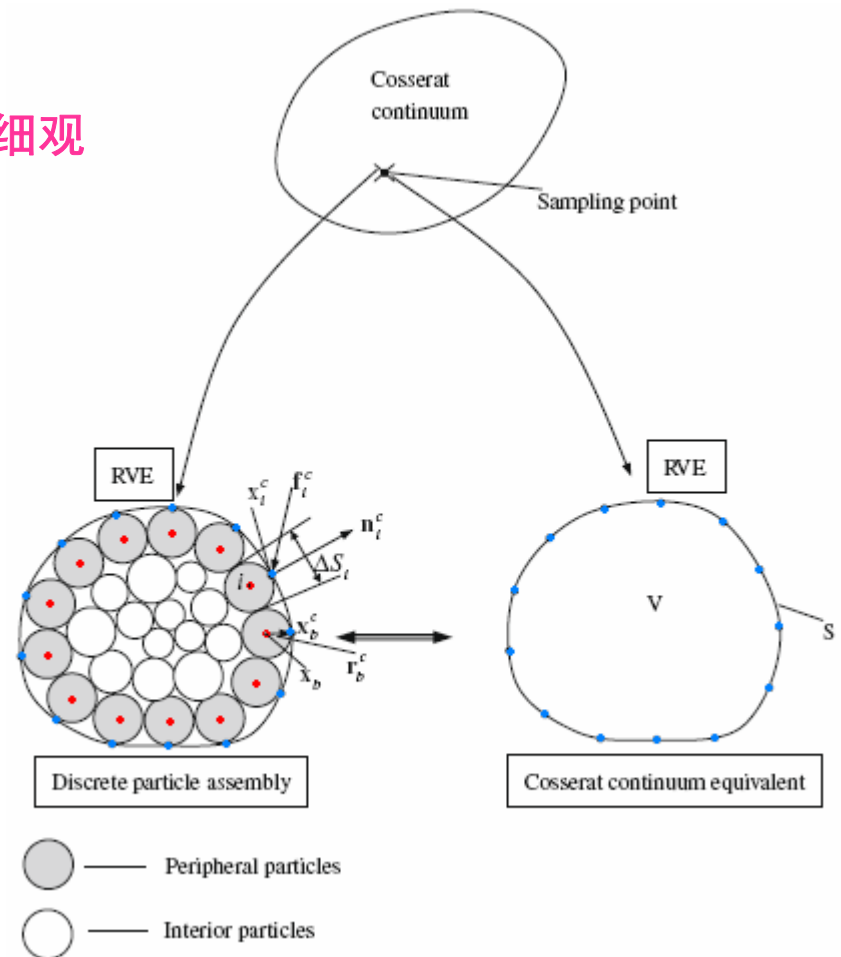
$$\frac{1}{V} \int_S (n_k \mu_{ki} - n_k \overline{\mu}_{ki}) (\omega_i - \overline{\omega}_i) dS$$

如何保证宏-细观  
能量等价?

RVE边界积分项

Xikui Li, Qipeng Liu. A version of Hill's lemma for Cosserat continuum, *Acta Mechanica Sinica*, 2009, 25:499-506.

Xikui Li, Qipeng Liu, Junbo Zhang. A micro-macro homogenization approach for discrete particle assembly - Cosserat continuum modeling of granular material, *Int. J. Solids and Structures*, 2010, 47:291-303.



# 离散颗粒集合体 - 梯度增强 Cosserat 连续体模型的细-宏观均匀化方法

- 导出基于细观微结构和细观响应量的宏观本构方程 (IJSS, 2010)

$$\begin{aligned} \dot{\boldsymbol{\sigma}} &= \mathbf{D}_{\sigma E} : \dot{\mathbf{E}} + \mathbf{D}_{\sigma \kappa} : \dot{\boldsymbol{\kappa}} \\ \dot{\boldsymbol{\mu}} &= \mathbf{D}_{\mu E} : \dot{\mathbf{E}} + \mathbf{D}_{\mu \kappa} : \dot{\boldsymbol{\kappa}} \end{aligned}$$

宏观应变度量

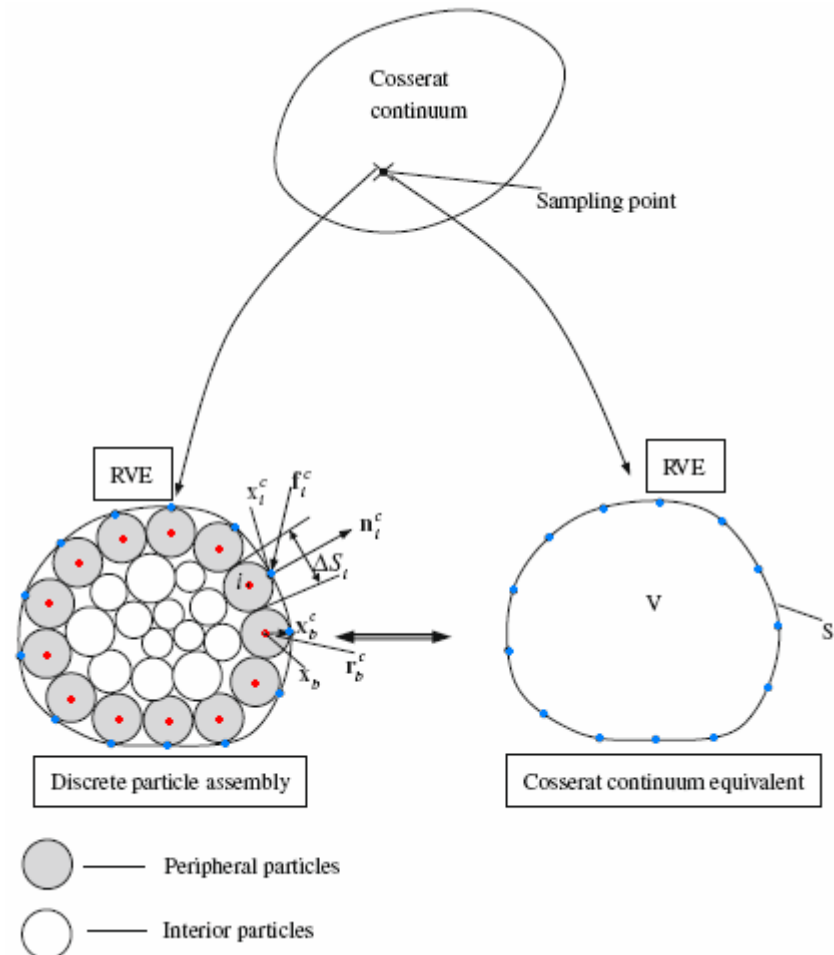
$$\dot{\mathbf{E}} = \dot{E}_{ji} = \dot{u}_{i,j} \quad \dot{\boldsymbol{\kappa}} = \dot{\kappa}_{ji} = \dot{\omega}_{i,j}$$

宏观应力度量

$$\boldsymbol{\sigma}, \boldsymbol{\mu}$$

基于细观微结构和细观响应量的宏观切线刚度模量张量

$$\mathbf{D}_{\sigma E}, \mathbf{D}_{\sigma \kappa}, \mathbf{D}_{\mu E}, \mathbf{D}_{\mu \kappa}$$



Xikui Li, Qipeng Liu, Junbo Zhang. A micro-macro homogenization approach for discrete particle assembly - Cosserat continuum modeling of granular material, *Int. J. Solids and Structures*, 2010, 47:291-303.

# 离散颗粒集合体 - 梯度增强Cosserat连续体模型的细-宏观均匀化方法

■ 提出了非均质梯度Cosserat连续体的广义Hill定理。

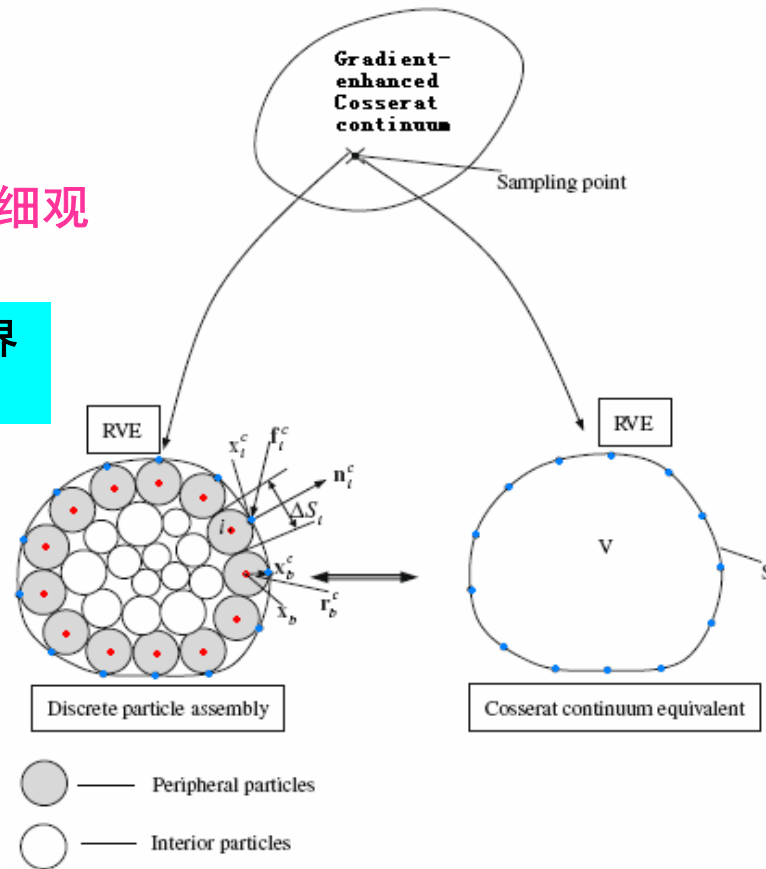
细观应变能

宏观应变能

如何保证宏-细观能量等价?

$$\begin{aligned} & \overline{\sigma_{ji} \varepsilon_{ji}} + \overline{\mu_{ji} \kappa_{ji}} - \overline{\bar{\sigma}_{ji} \bar{\varepsilon}_{ji}} - \overline{\bar{\mu}_{ji} \bar{\kappa}_{ji}} - \overline{\Sigma_{lji} \bar{E}_{lji}} \\ &= \frac{1}{V} \int_S (n_k \sigma_{ki} - n_k \bar{\sigma}_{ki}) (u_i - \bar{u}_{i,j} x_j - \frac{1}{2} \bar{u}_{i,jl} x_j x_l) dS \\ &+ \frac{1}{V} \int_S (n_k \mu_{ki} - n_k \bar{\mu}_{ki}) (\omega_i - \bar{\omega}_i - \bar{\omega}_{i,l} x_l) dS \end{aligned}$$

RVE边界积分项



Xikui Li, Junbo Zhang, Xue Zhang.  
Micro-macro homogenization of gradient-enhanced  
Cosserat media, **European Journal of Mechanics  
A/Solids**. 2011, 30: 362-372.

问题的提出: 为什么要考虑  
梯度增强Cosserat连续体模型?

- RVE的有限尺度和计及尺寸效应
- 考虑处于高应变区域的RVE

# 离散颗粒集合体 - 梯度增强Cosserat连续体模型 的细-宏观均匀化方法

- 如何保证非均匀梯度**Cosserat**连续体多尺度分析中宏细观能量等价、即Hill-Mandel能量条件成立？

细观应变能

宏观应变能

$$\overline{\sigma_{ji} \varepsilon_{ji}} + \overline{\mu_{ji} \kappa_{ji}} = \overline{\sigma_{ji}} \overline{\Gamma_{ji}} + \overline{T_k} \overline{\omega_k} + \overline{\mu_{ji}^0} \overline{\kappa_{ji}} + \widehat{\Sigma}_{jlk} \overline{E}_{jlk}$$

要求满足如下边界条件

$$\frac{1}{V} \int_S (n_k \sigma_{ki} - n_k \overline{\sigma}_{ki}) (u_i - \overline{u}_{i,j} x_j - \frac{1}{2} \overline{u}_{i,jl} x_j x_l) dS$$

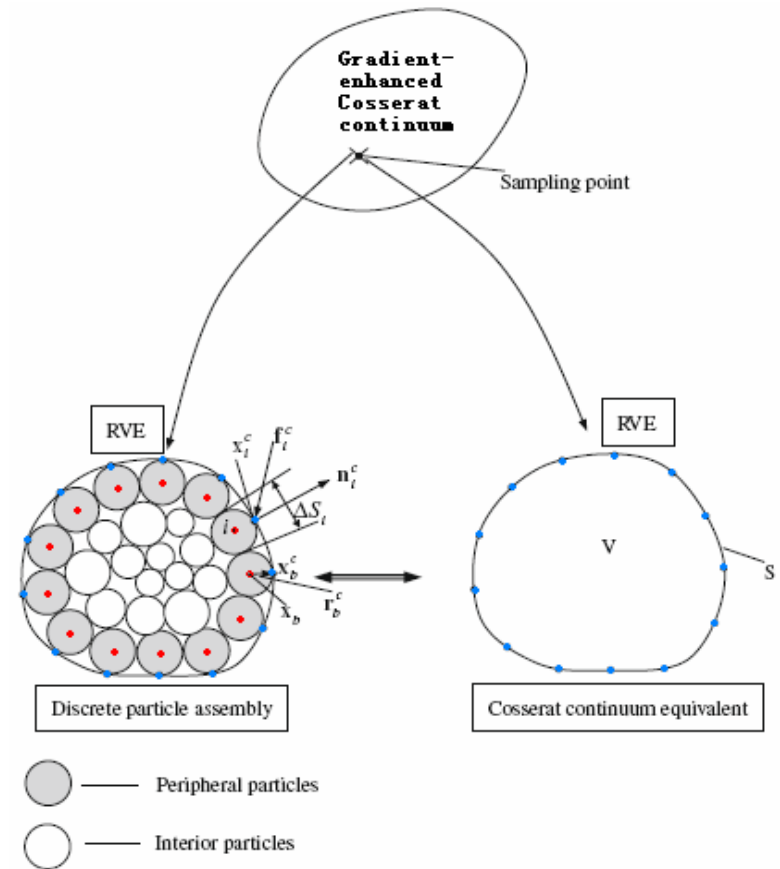
$$+ \frac{1}{V} \int_S (n_k \mu_{ki} - n_k \overline{\mu}_{ki}) (\omega_i - \overline{\omega}_i - \overline{\omega}_{i,l} x_l) dS = 0$$

由此导出**RVE**边界条件 - 一致性线-角位移边界条件



$$\begin{cases} u_i|_S = \overline{u}_{i,j} x_j + \frac{1}{2} \overline{u}_{i,jl} x_j x_l \\ \omega_i|_S = \overline{\omega}_i + \overline{\omega}_{i,l} x_l \end{cases}$$

或一致性位移边界条件  
+ 周期性边界条件



# 离散颗粒集合体 - 梯度增强Cosserat连续体模型的细-宏观均匀化方法

■ 发展了基于细观力学信息的模拟颗粒材料力学响应的离散颗粒集合体-(梯度增强)Cosserat连续体的宏观本构模型与细宏观计算均匀化方法。

重表示宏观应变能, Hill-Mandel能量条件

$$\bar{\sigma}_{ji} \bar{\varepsilon}_{ji} + \bar{\mu}_{ji} \bar{\kappa}_{ji} + \bar{\Sigma}_{lji} \bar{E}_{lji} = \bar{\sigma}_{ji} \bar{\Gamma}_{ji} + \bar{T}_k \bar{\omega}_k + \bar{\mu}_{ji}^0 \bar{\kappa}_{ji} + \hat{\Sigma}_{jlk} \bar{E}_{jlk}$$

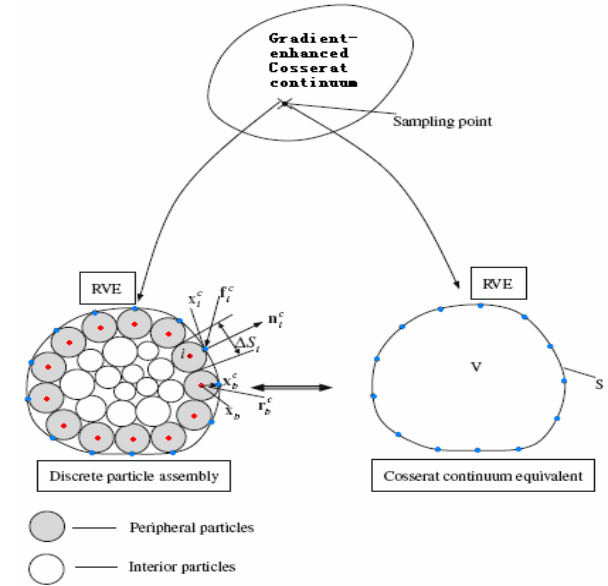
• 基于细观微结构和细观响应量的宏观切线刚度模量张量

## 梯度Cosserat连续体

$$\begin{aligned} \dot{\bar{\sigma}} &= \mathbf{D}_{\sigma\Gamma} : \dot{\bar{\Gamma}} + \mathbf{D}_{\sigma E} : \dot{\bar{E}} + \mathbf{D}_{\sigma\omega} \cdot \dot{\bar{\omega}} + \mathbf{D}_{\sigma\kappa} : \dot{\bar{\kappa}} \\ \dot{\hat{\Sigma}} &= \mathbf{D}_{\hat{\Sigma}\Gamma} : \dot{\bar{\Gamma}} + \mathbf{D}_{\hat{\Sigma}E} : \dot{\bar{E}} + \mathbf{D}_{\hat{\Sigma}\omega} \cdot \dot{\bar{\omega}} + \mathbf{D}_{\hat{\Sigma}\kappa} : \dot{\bar{\kappa}} \\ \dot{\bar{T}} &= \mathbf{D}_{T\Gamma} : \dot{\bar{\Gamma}} + \mathbf{D}_{TE} : \dot{\bar{E}} + \mathbf{D}_{T\omega} \cdot \dot{\bar{\omega}} + \mathbf{D}_{T\kappa} : \dot{\bar{\kappa}} \\ \dot{\bar{\mu}}^0 &= \mathbf{D}_{\mu\Gamma} : \dot{\bar{\Gamma}} + \mathbf{D}_{\mu E} : \dot{\bar{E}} + \mathbf{D}_{\mu\omega} \cdot \dot{\bar{\omega}} + \mathbf{D}_{\mu\kappa} : \dot{\bar{\kappa}} \end{aligned}$$

Xikui Li et al. A generalized Hill's lemma and micromechanically based macroscopic constitutive model for heterogeneous granular materials

Computer Methods in Appl. Mech. Eng., 199: 3137-3152, 2010.



## 经典Cosserat连续体

$$\begin{aligned} \dot{\bar{\sigma}} &= \mathbf{D}_{\sigma E} : \dot{\bar{E}} + \mathbf{D}_{\sigma\kappa} : \dot{\bar{\kappa}} \\ \dot{\bar{\mu}} &= \mathbf{D}_{\mu E} : \dot{\bar{E}} + \mathbf{D}_{\mu\kappa} : \dot{\bar{\kappa}} \end{aligned}$$

Xikui Li et al. A micro-macro homogenization approach for discrete particle assembly - Cosserat continuum modeling of granular material, Int. J. Solids and Struct, 2010, 47: 291-303.

- 考虑微结构演变及材料几何非线性
- 无需宏观材料参数和宏观本构模型

# 离散颗粒集合体 - Cosserat连续体模型 的连接尺度方法

- 发展了耦合离散单元法(DEM)和Cosserat连续体有限元法的连接尺度方法

优缺点比较

细观尺度模拟

宏观尺度模拟

	离散元法 (DEM)	Cosserat 连续体有限元法
显示微结构演变	能	不能
揭示微观破坏机理	能	不能
计算成本	高	低
存贮空间	大	小

BSM 结合了细-宏观尺度模拟的优点

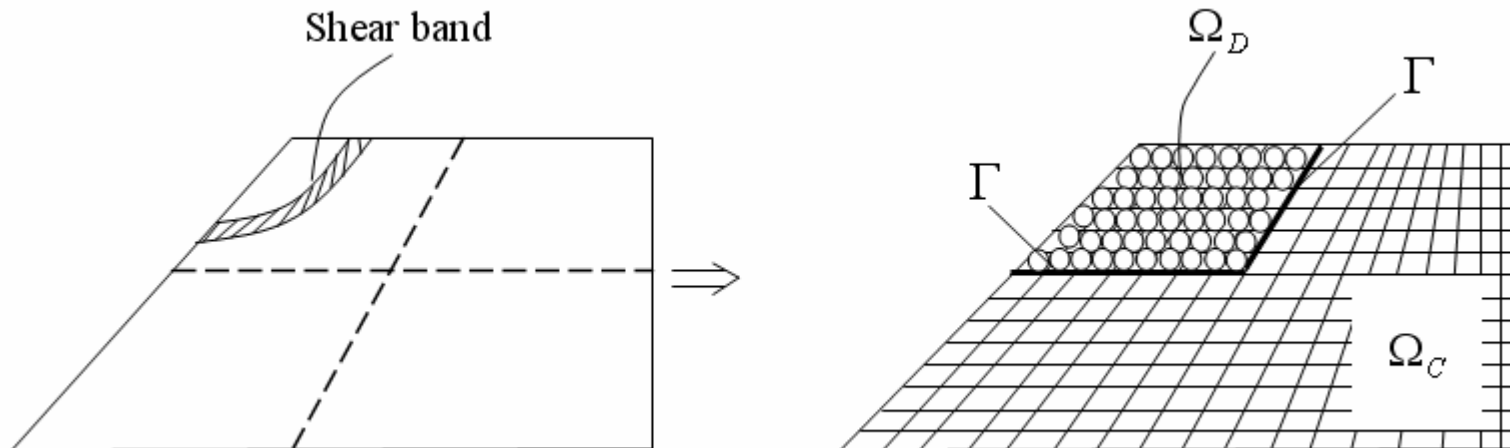
# 离散颗粒集合体 - Cosserat连续体模型 的连接尺度方法

- 仅对局部区域模型化为离散颗粒集合体和利用离散元法(DEM)求解

- 全域模型化为Cosserat连续体和利用有限元法(FEM)离散求解.

- 允许在DEM和FEM域中采用不同时间步长.

- Greatly enhance computational efficiency
- Able to capture micromechanically- based failure mechanism and to perform discontinuous analysis

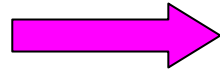


# 离散颗粒集合体 - Cosserat连续体模型 的连接尺度方法

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现有 **BSM**

(by Liu WK and G. J. Wagner,  
2003,2005)



(extension)

本工作 **BSM**

纳米材料

颗粒材料

**Cauchy** 连续体  
(粗尺度)

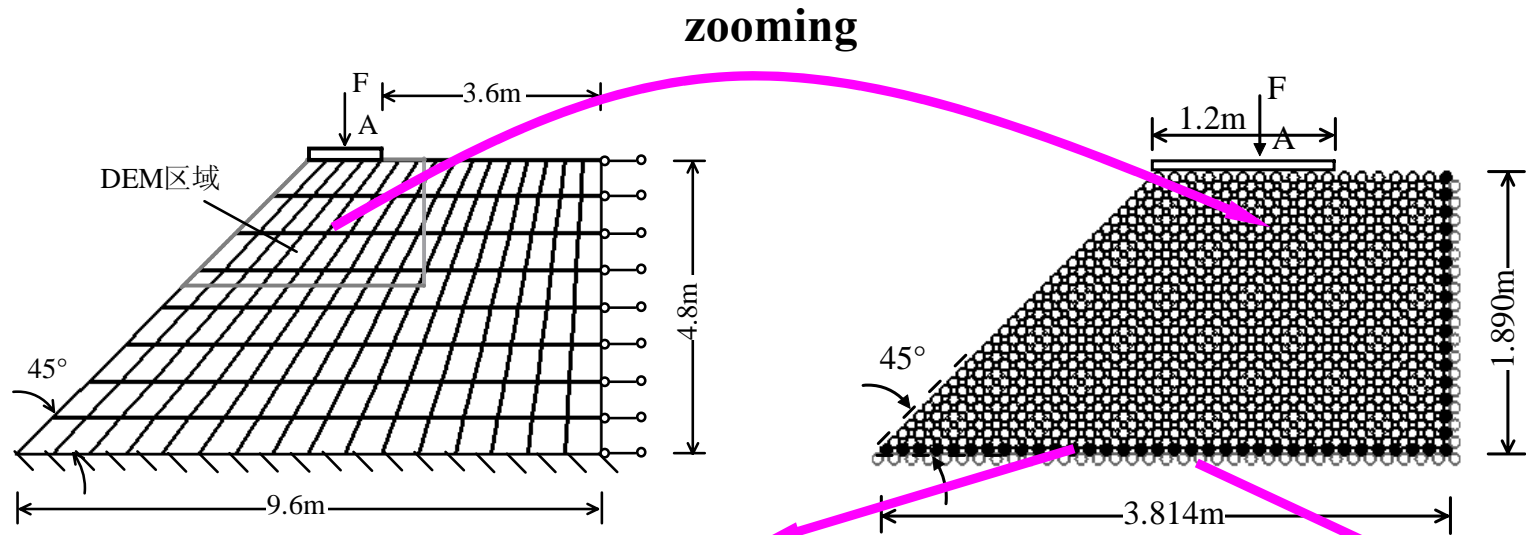
**Cosserat** 连续体  
(粗尺度)

分子动力学模型  
(细尺度)

离散颗粒集合体模型  
(细尺度)



# 离散颗粒集合体 - Cosserat连续体模型 的连接尺度方法



DEM域中（与粗尺度域）  
界面颗粒

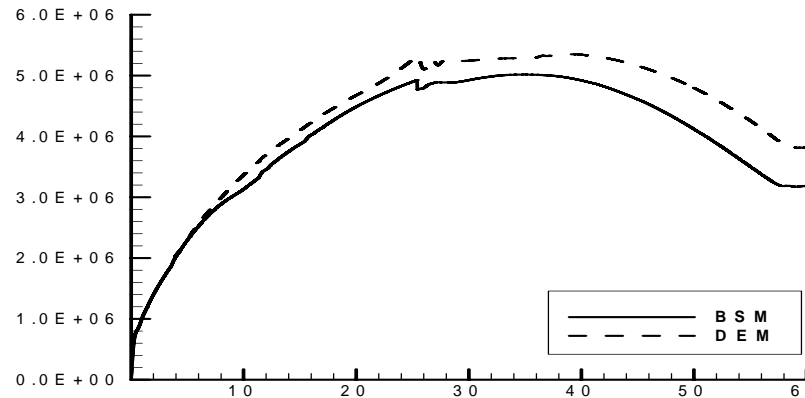
粗尺度域中（与DEM  
域毗邻）虚拟颗粒

- 在界面颗粒与虚拟颗粒集合体处设计了在动态载荷条件下可以有效消除粗细尺度域界面上虚假反射波和数值震荡的非反射边界条件
- 提出了在拟静态载荷条件下高效的粗细尺度界面条件

• Xikui Li, Ke Wan, A bridging scale method for granular materials with discrete particle assembly - Cosserat continuum modeling, submitted.

# 离散颗粒集合体 - Cosserat连续体模型 的连接尺度方法

边坡稳定问题结果



BSM和DEM计算  
所得的边坡位移  
承载曲线比较

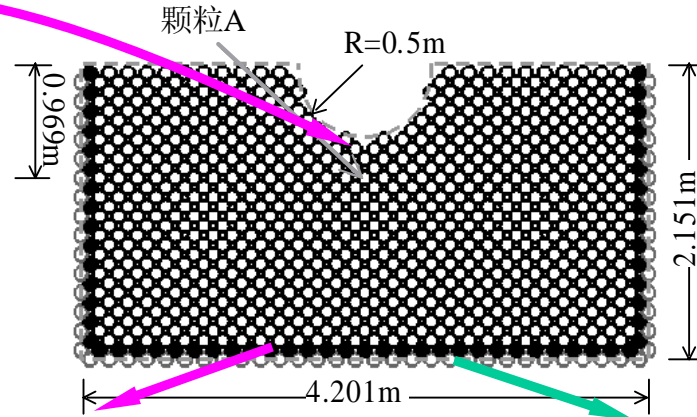
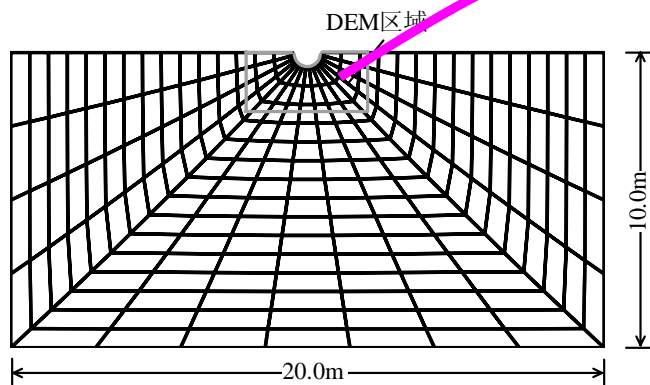
	BSM	DEM	差值百分比
垂直荷载峰值 (KN)	4993.31	5353.74	-6.73%
最小体积应力 (MPa)	8.3740	9.1955	-8.93%
最大有效应力 (MPa)	23.0792	23.3921	-1.34%
最小体积应变	-0.09185	-0.08664	6.01%
最大有效应变	0.7367	0.7711	-4.46%
最大颗粒转角(rad)	0.9665	0.9899	-2.36%
最小颗粒转角 (rad)	-1.0251	-1.1360	-9.76%
总计算时间(s)	2146.25	55922.75	-96.16%

DEM和BSM计算结果与计算效率比较

# 离散颗粒集合体 - Cosserat连续体模型 的连接尺度方法

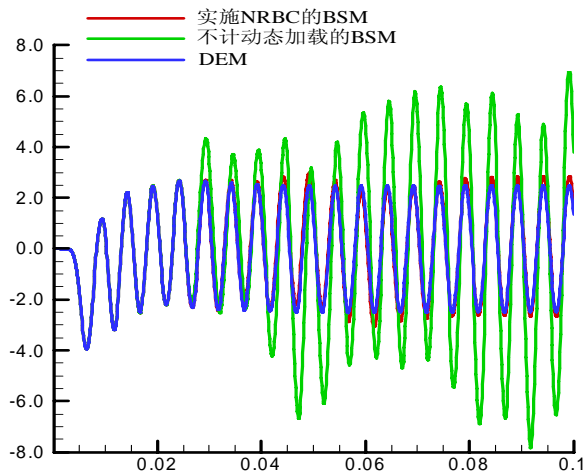
隧道问题：非反射边界条件效应

隧道表面压力荷载历史  $q_n = q_{n,0} \cos(\omega t)$

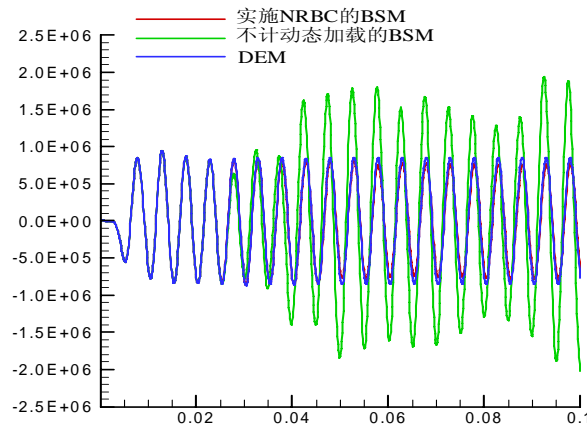


DEM域中（与粗尺度域）界面颗粒

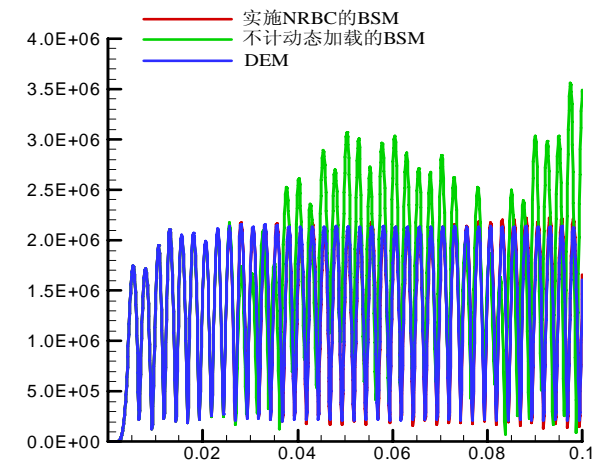
粗尺度域中（与DEM域毗邻）虚拟颗粒



颗粒A的径向位移演化



颗粒A的体积应力演化



颗粒A的有效应力演化

谢谢!